Online Computation with Machine-Learned Advice

Spyros Angelopoulos and Christoph Dürr
CNRS and LIP6-Sorbonne University

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Background

In online computation we are interested in algorithms that do not know the entire input in advance, but rather process it in the form of a sequence of input items. For each incoming input item, the algorithm must make an irrevocable decision concerning it, in general without any assumptions on the future items. Problems of this type arise very often in real-world applications, such as routing calls within a communications network, managing the cache content of web servers, or deciding how to auction goods among a number of participating agents. For this reason, online computation has developed, over the past few decades, into a prolific field with significant impact in many areas of Computer Science, and in particular in Theoretical Computer Science (TCS), Operations Research (OR), Artificial Intelligence (AI) and Machine Learning (ML). See for example the books [9,15], and a new book in preparation [10], which is a testament to the ever-increasing interest in the field.

How does one analyze and compare the performance of online algorithms? The standard method of evaluation is by means of the *competitive analysis* framework: here, we compare the worst-case performance of the online algorithm to the corresponding performance of an ideal, offline algorithm, namely an algorithm that has advanced knowledge of the entire input. Thus, a good online algorithm guarantees a small gap between the online and the ideal (offline) performance; this measure is called the *competitive ratio* of the algorithm.

General presentation of the topic

The standard model of online computation assumes that the online algorithm has no knowledge whatsoever about the input. This can be problematic since, in the real world, the algorithm may indeed have such (limited) knowledge. For example, in the context of routing calls in a network, we may have information on the participants that are likely to call each other, or the periods of the day during which the majority of the calls are placed. How can then one adapt the main online paradigm to account for situations such as the above? There have been two main approaches:

1. *The advice-complexity framework*. Here, we assume that the online algorithm has access to an oracle that provides the additional information, termed *advice*. More precisely, in the advice setting, the online algorithm receives some bits that encode information concerning the sequence of input items. This additional information can help improve the competitive ratio of the algorithm. It is important to note that this advice is assumed to be i) error-free; and ii) provided by an omnipotent, and potentially unrealistic oracle. This is an area that has been studied for over a decade within theoretical computer science, see e.g the survey [11] and the book [15].

2. *The ML-prediction framework.* This is a much more recent approach that is rooted in machine learning. Here, the additional information is in the form of a natural prediction concerning the input; for example, the point in time at which certain items will appear. It is important to note that the prediction is assumed to be i) potentially noisy, that is, there is some *error* associated with the

prediction; ii) provided by a "reasonable" and "natural" oracle, i.e., an oracle that provides some information on the input that can be easily learnable.

It should be clear that the two approaches are almost complementary, and one would dare say, in conflict. The advice-complexity framework is almost entirely a theoretical abstraction with very few applications, but allows for very rigorous analysis using information-theoretic approaches. In contrast, the more recent ML-prediction framework is motivated by real applications, but has not yet fully developed the precise theory about the limitations of predictions.

Objectives of this thesis proposal

The main objective in this project can be summarized as follows:

Can we narrow the big gap between the advice-complexity model and the ML-prediction model? Can we obtain better online algorithms with predictions by combining, and extending techniques from both approaches?

Below we present some specific directions we will pursue with the PhD student.

1. *The price of erroneous advice.* In our work [1] and [2] we studied advice in a setting in which it is either always correct, or always adversarially generated (informally: either provided by a friend or by an enemy). For this reason, some of the performance tradeoffs are only of theoretical significance. A more realistic model would allow for *noisy* advice (see [5] for a recent application in online financial optimization). This can be interpreted as using a number of noisy *binary queries* as predictions. There is a plethora of problems to be studied in this setting.

2. *Compact predictions and learnability*. The ML-prediction framework so far has not placed any restrictions on the size of the advice, unlike the advice-complexity framework. However, there are situations in which some restriction is necessary in practice (e.g., in caching, we cannot afford too much time in gathering information). Thus, we would like to study *time-sensitive* problems and applications (e.g., caching and its generalizations) in a setting in which there is a prediction of small size. On the flip side, the advice-complexity framework does not consider the learnability of the advice, hence we would like to study the power and limitations of *learnable* advice.

3. *Combining advice from several experts*. Here, we are interested in using potentially erroneous predictions that may originate from several sources (i.e., experts), instead of a single one. This was only recently explored in [13] for the ski rental problem, and the same question can be asked for many other online problems.

4. *Better tools for analysis of algorithms with predictions.* The advice complexity field has introduced some powerful theoretical tools (e.g., the string guessing game [12]), which however are tailored to a setting in which the advice is error-free. We would like to extend them to a more general setting in which the advice is imperfect.

The overall approach to these problems will involve two steps. The first is the theoretical analysis of the online algorithms and their limitations, from the point of view of both positive and negative (i.e., impossibility) results. The second step is the experimental validation of these algorithms.

State of the art and global positioning of this project

This line of research was introduced in [17,20] and combines aspects of theoretical computer science and machine learning. These works proved very influential, are already heavily cited, and spearheaded an impressive activity. Further applications include secretary and matching problems

[7], page migration [14], ski rental and rent-or-buy problems [8,13], metrical task systems [6], scheduling [16] and many other problems.

The scope of this project lies within a broader objective related to the analysis of algorithms beyond the worst-case. Here, the emphasis is on assessing algorithms not only with respect to the pessimistic worst-case, but using refined measures that better reflect the typical algorithmic performance. This is an emerging trend that became even more attractive with the publication of a book by Roughgarder on this topic (see, in particular, the chapter by Mitzenmacher and Vassilvitskii on algorithms with predictions in this book [18]).

This line of research is already receiving considerable exposure, with presentations in venues of high visibility (e.g., the talk by Mitzenmacher at the Simons Institute and the talk by Vassilvitskii at Highlights of Algorithms 2019). Related topics have been presented at workshops such as the Workshop on Algorithms with Predictions (ALPS 2022), the TTIC Summer Workshop on Learning-Based Algorithms, the STOC 2020 Workshop on Algorithms with Predictions, the 2020 Workshop on Online Algorithms with Advice and Related Models and courses such as the MIT course on Learning-Augmented Algorithms.

Qualifications and related publications by the supervisor

The supervisors of the project have several publications in leading TCS conferences and journals (JACM, SODA, ITCS, ESA, STACS) as well as in leading AI conferences and journals (AAAI, IJCAI, JAIR), and have already contributed several publications related to this new area, e.g [1,2,3,4,5]. They have also given invited lectures at specialized workshops (e.g. the ALPS 2022 workshop).

Student profile

- M.Sc. degree in Computer Science or related discipline such as Operations Research.
- Strong background in the design and theoretical analysis of algorithms., as documented by previous courses or research experience.
- Programming skills and willingness to work with real data will be considered a plus.

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