Colloquium d'informatique de l'UPMC Sorbonne Universités

Abstract Interpretation

29 septembre 2016, 18:00, Amphi 15
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This is an abstract interpretation

Scientific research

Scientific research

- In Mathematics/Physics:

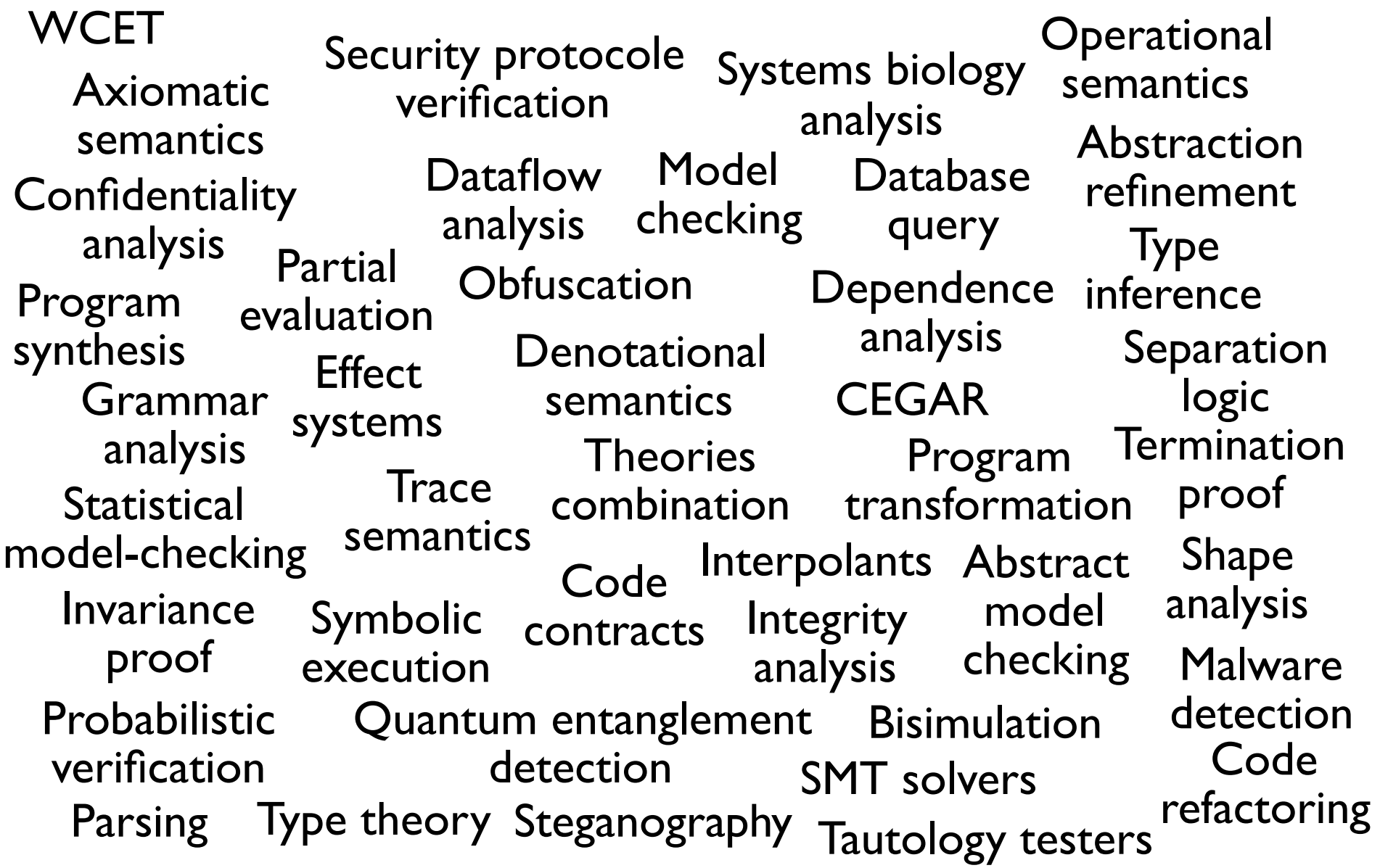
  trend towards unification and synthesis through universal principles

- In Computer science:

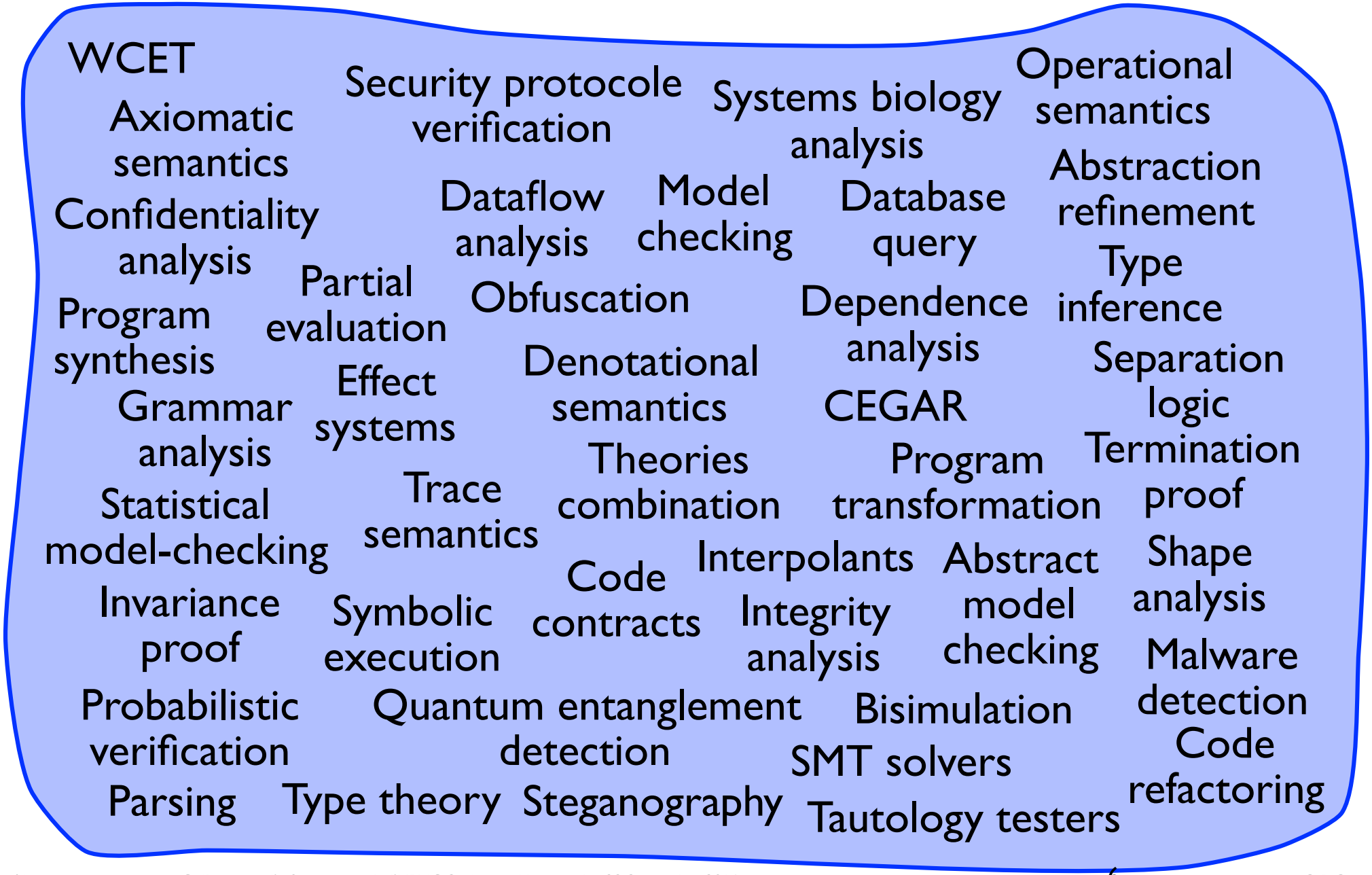
  trend towards dispersion and parcellation through a ever-growing collection of local ad-hoc techniques for specific applications

  An exponential process, will stop!

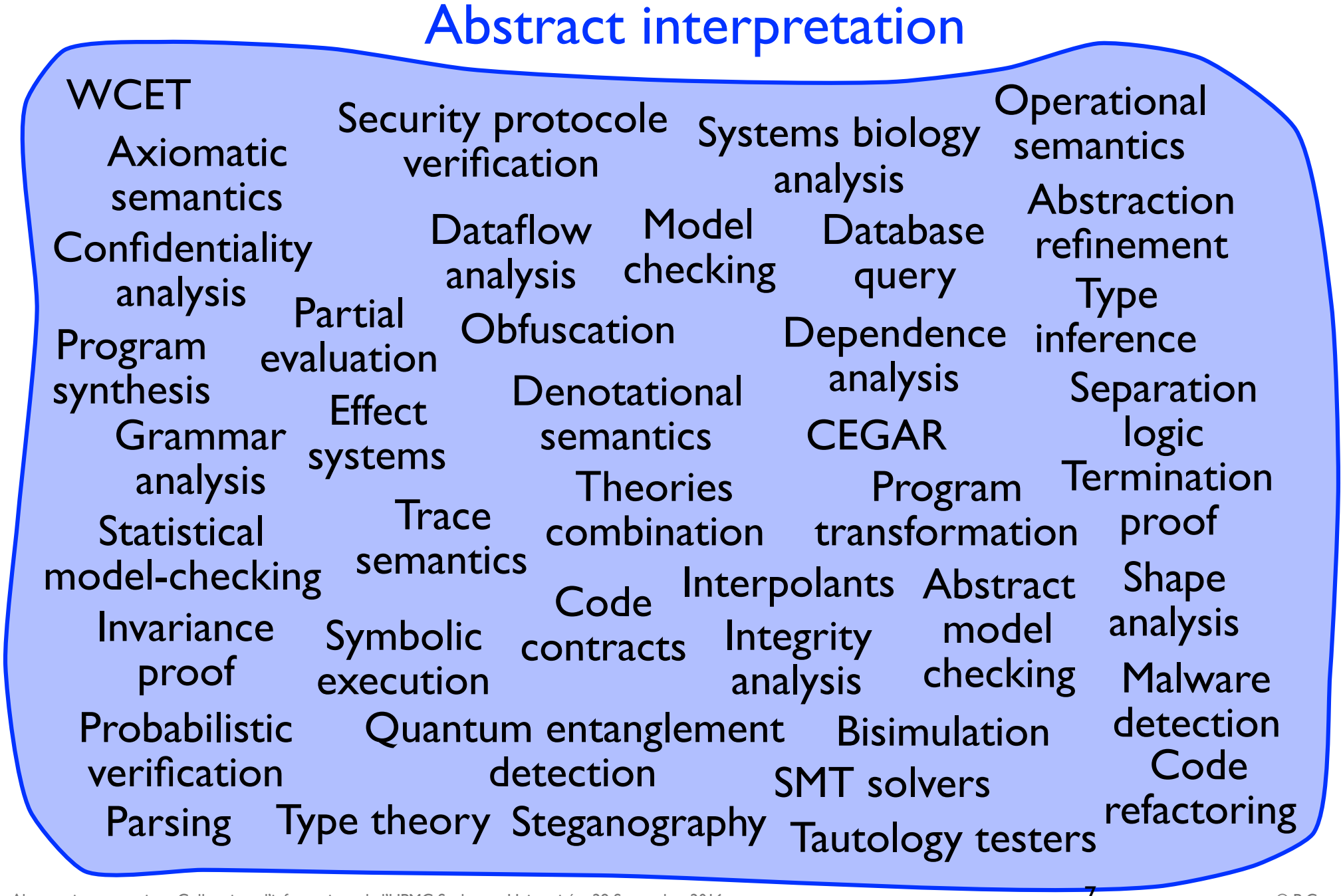
Example: reasoning on computational structures



Example: reasoning on computational structures



Example: reasoning on computational structures



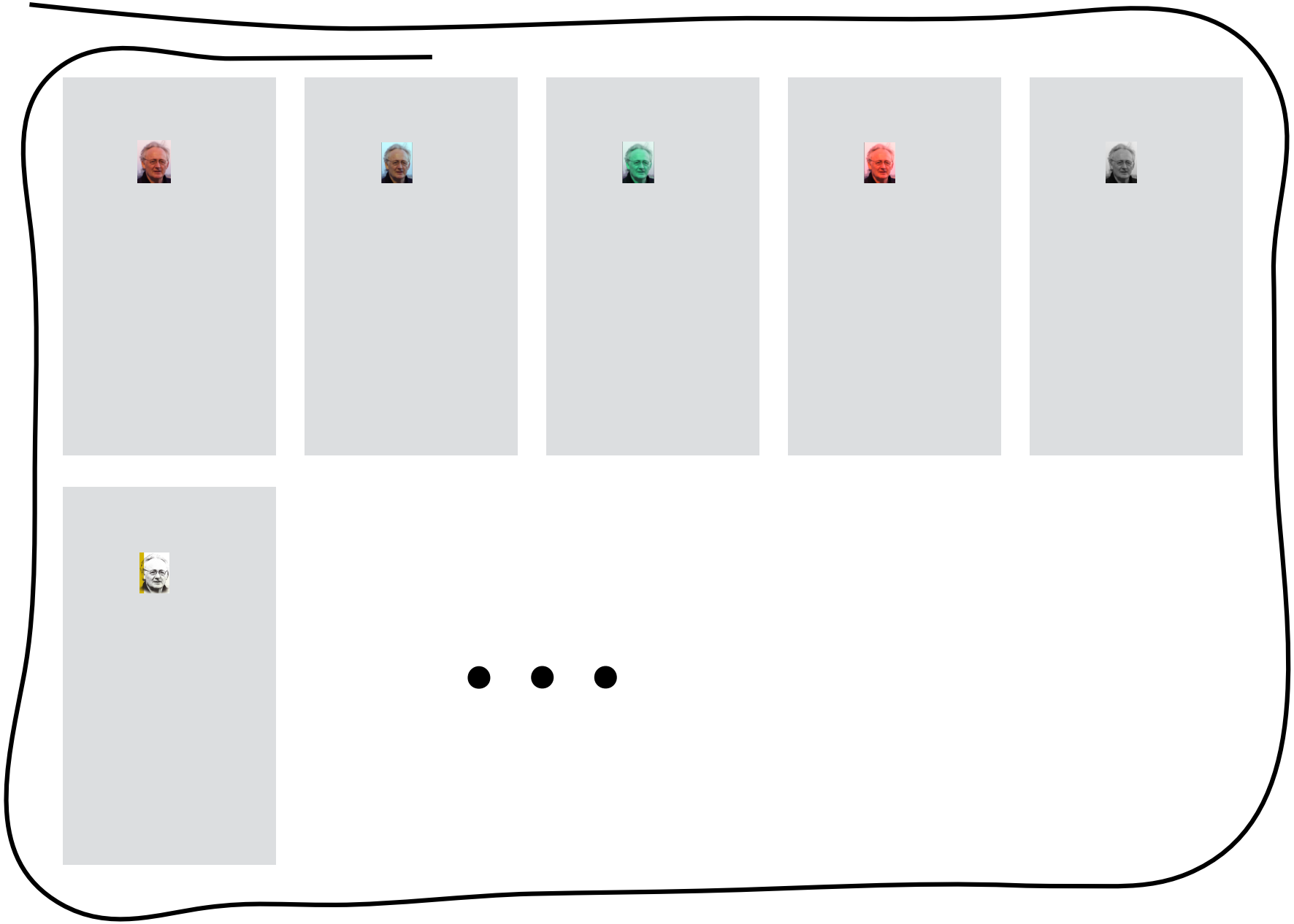
Intuition I

Concrete

Abstraction I

Abstraction 2

Concretization 2

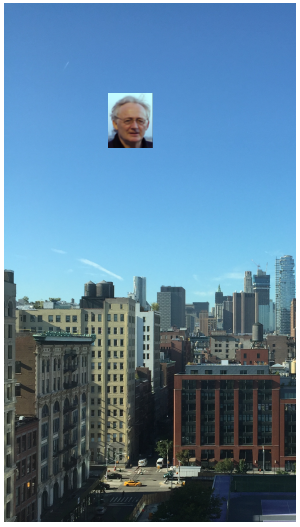


Concretization 1

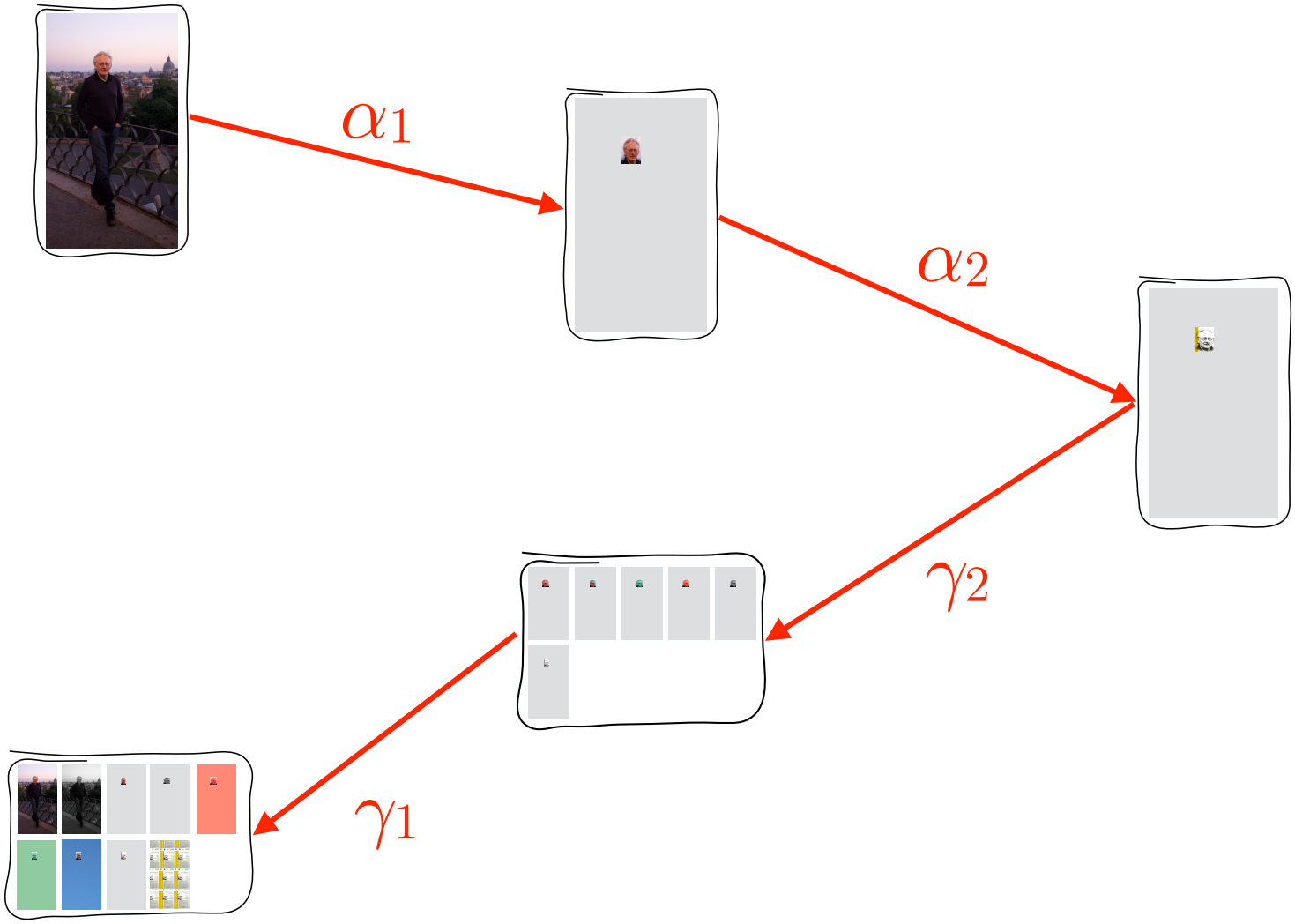




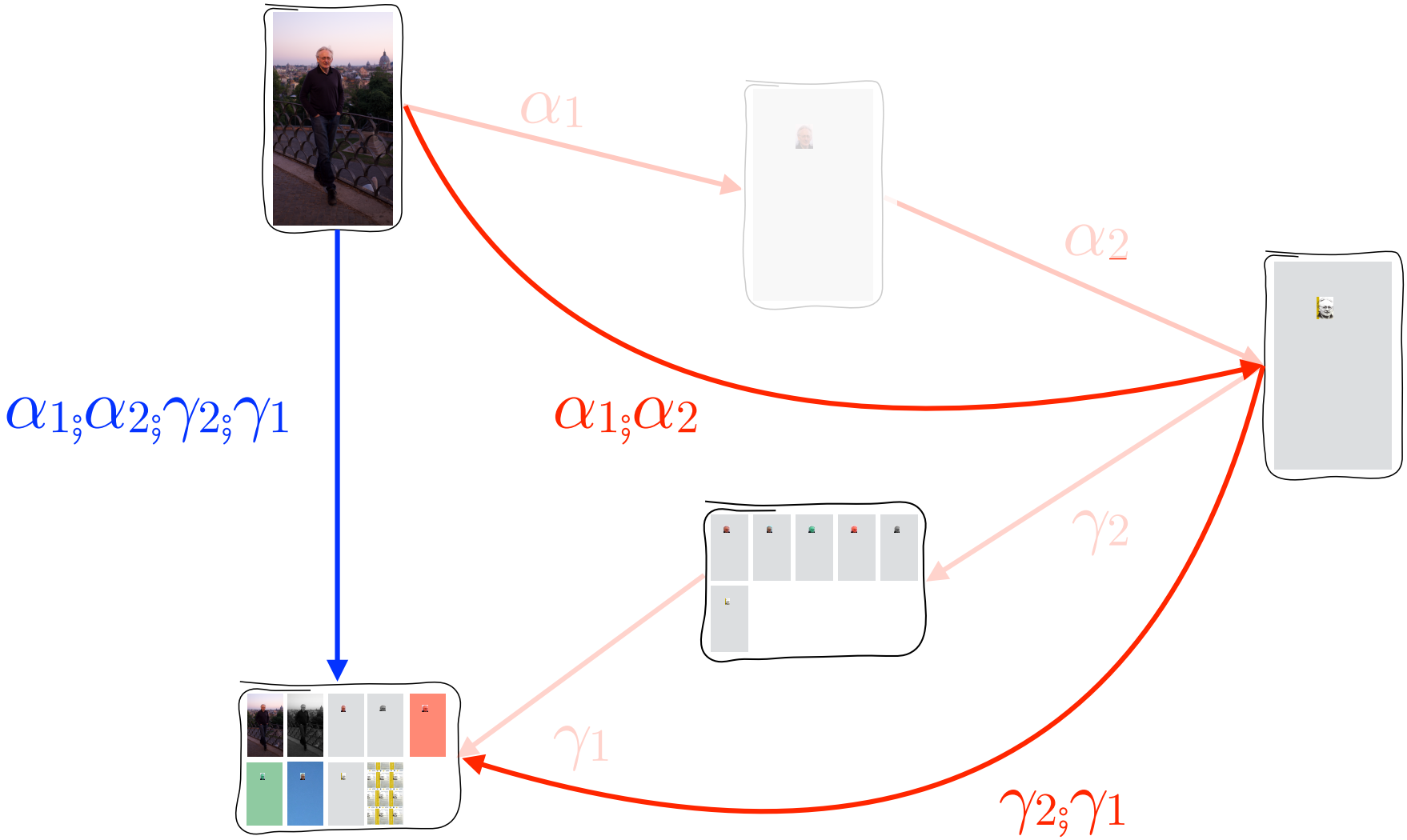




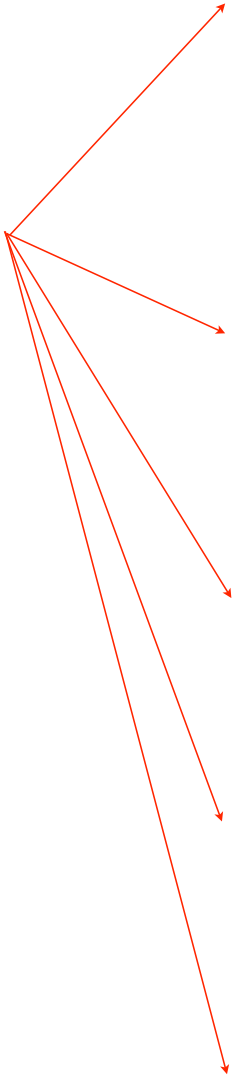

Abstract interpretations



Abstract interpretations



Intuition 2

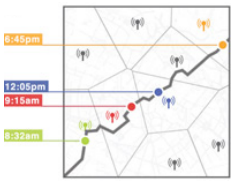


Height

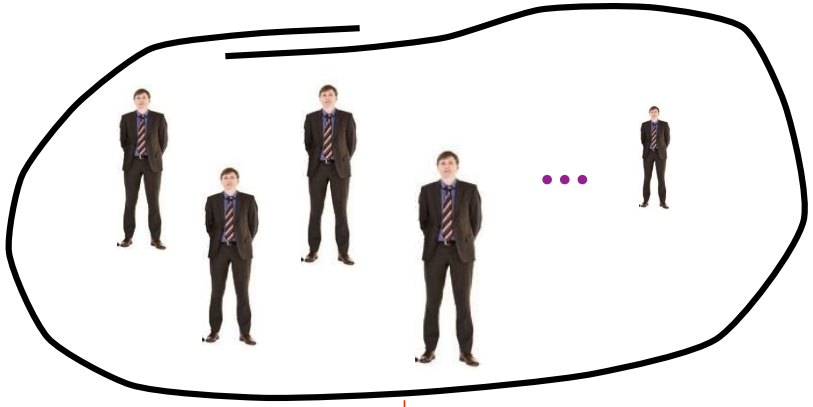
Fingerprint

Eye color

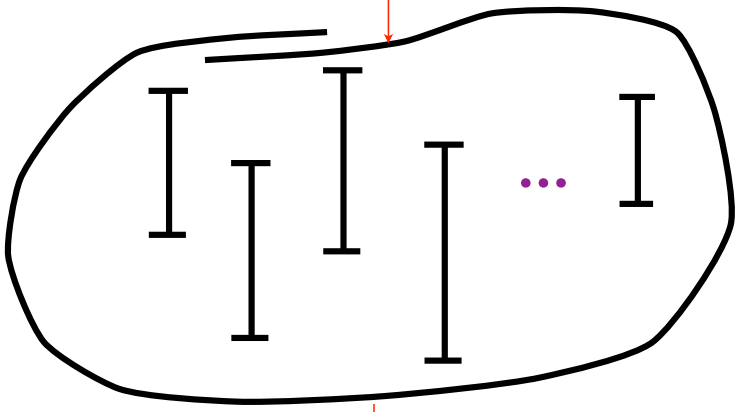
DNA



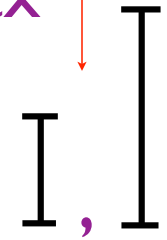
Phone metadata



Individual heights

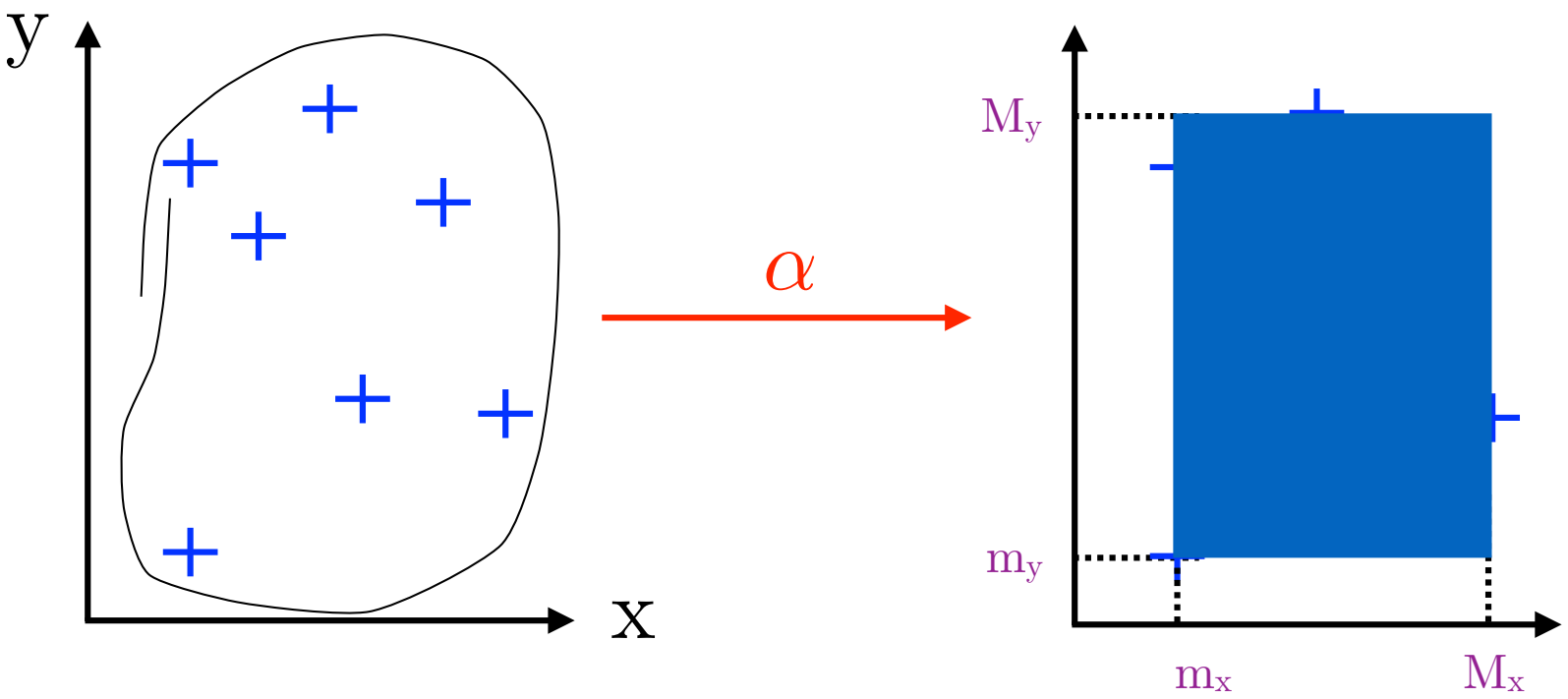


min, max



Interval abstraction

- Example: interval abstraction (also called *box abstraction*)



Set of points

Interval abstraction
$[m_x,M_x]\times[m_y,M_y]$

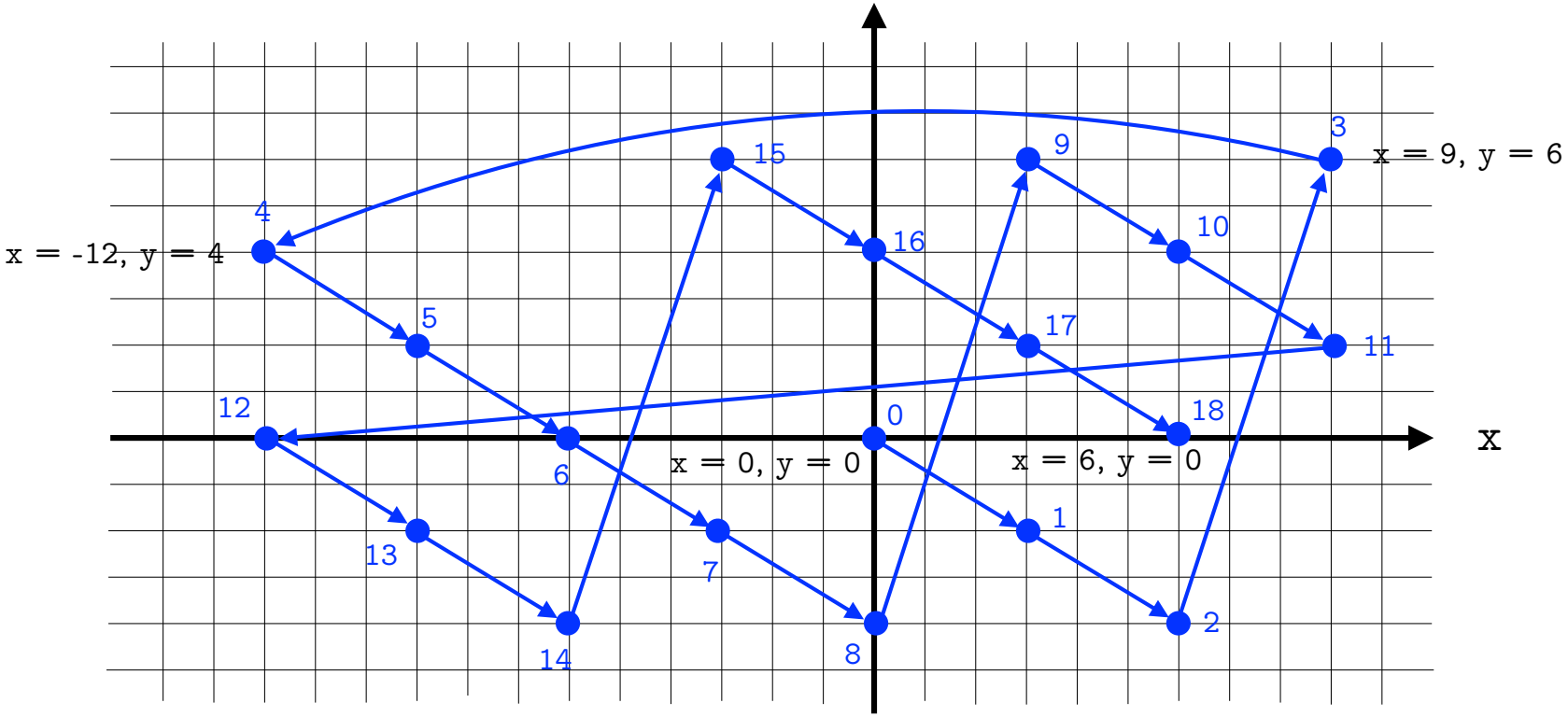
Intuition 3

A C program and one of its executions

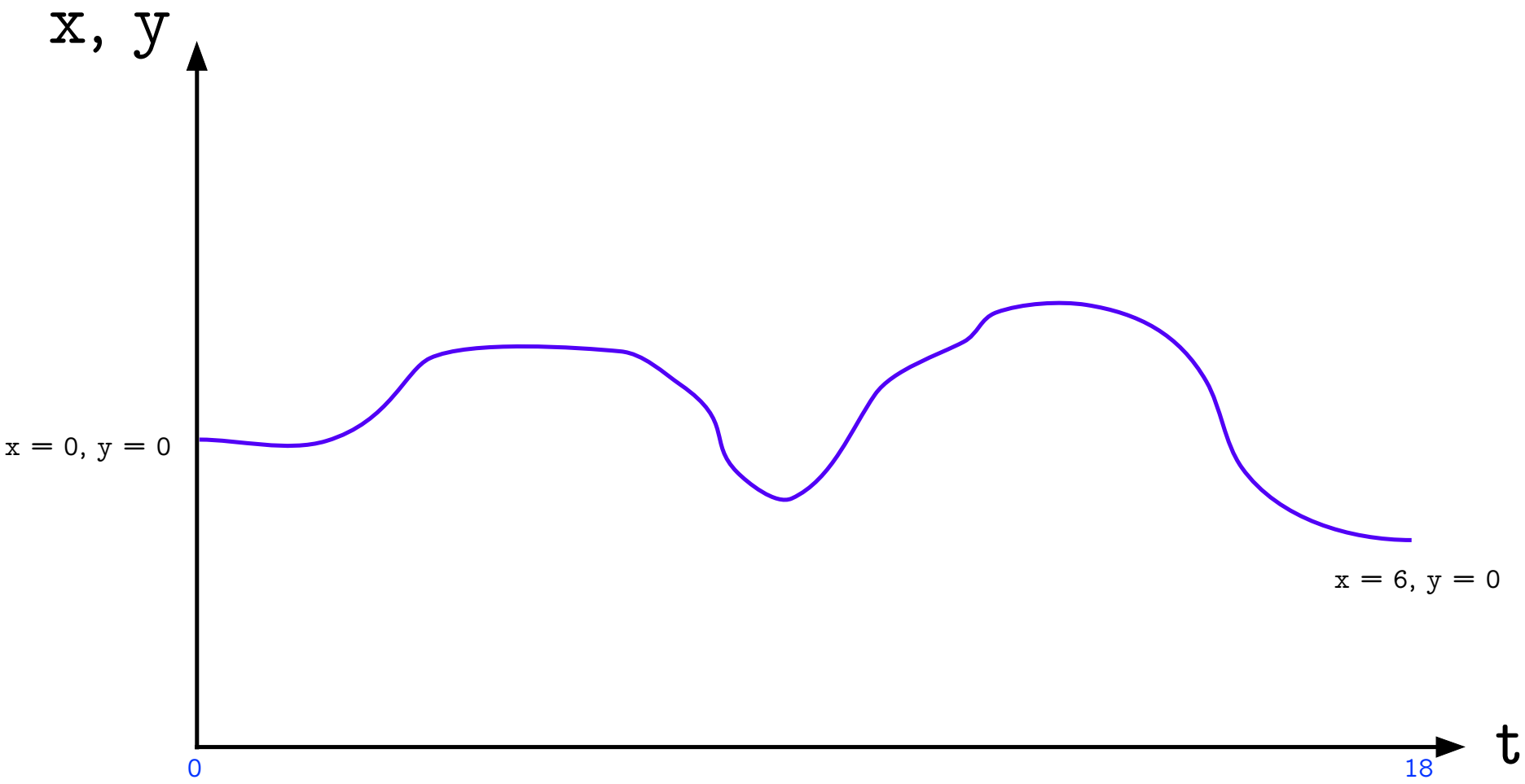
```
#include <stdio.h>
int main()
{
          int x, y;
          printf("Enter two integers: ");
          scanf("%d %d",&x, &y);
/* 1: */  while ((x != 6) || ( y != 0)) {
             printf("x = %d, y = %d\n",x,y);
/* 2: */     x = x + 3;
/* 3: */     if (x > 10) x = -x;
/* 4: */     y = y - 2;
/* 5: */     if (y < -5) y = -y;
          }
/* 6: */  printf("x = %d, y = %d\n",x,y);
}
```

```
Enter two integers: x = 0, y = 0
x = 3, y = -2
x = 6, y = -4
x = 9, y = 6
x = -12, y = 4
x = -9, y = 2
x = -6, y = 0
x = -3, y = -2
x = 0, y = -4
x = 3, y = 6
x = 6, y = 4
x = 9, y = 2
x = -12, y = 0
x = -9, y = -2
x = -6, y = -4
x = -3, y = 6
x = 0, y = 4
x = 3, y = 2
x = 6, y = 0
```

Graphical representation of the execution (I)

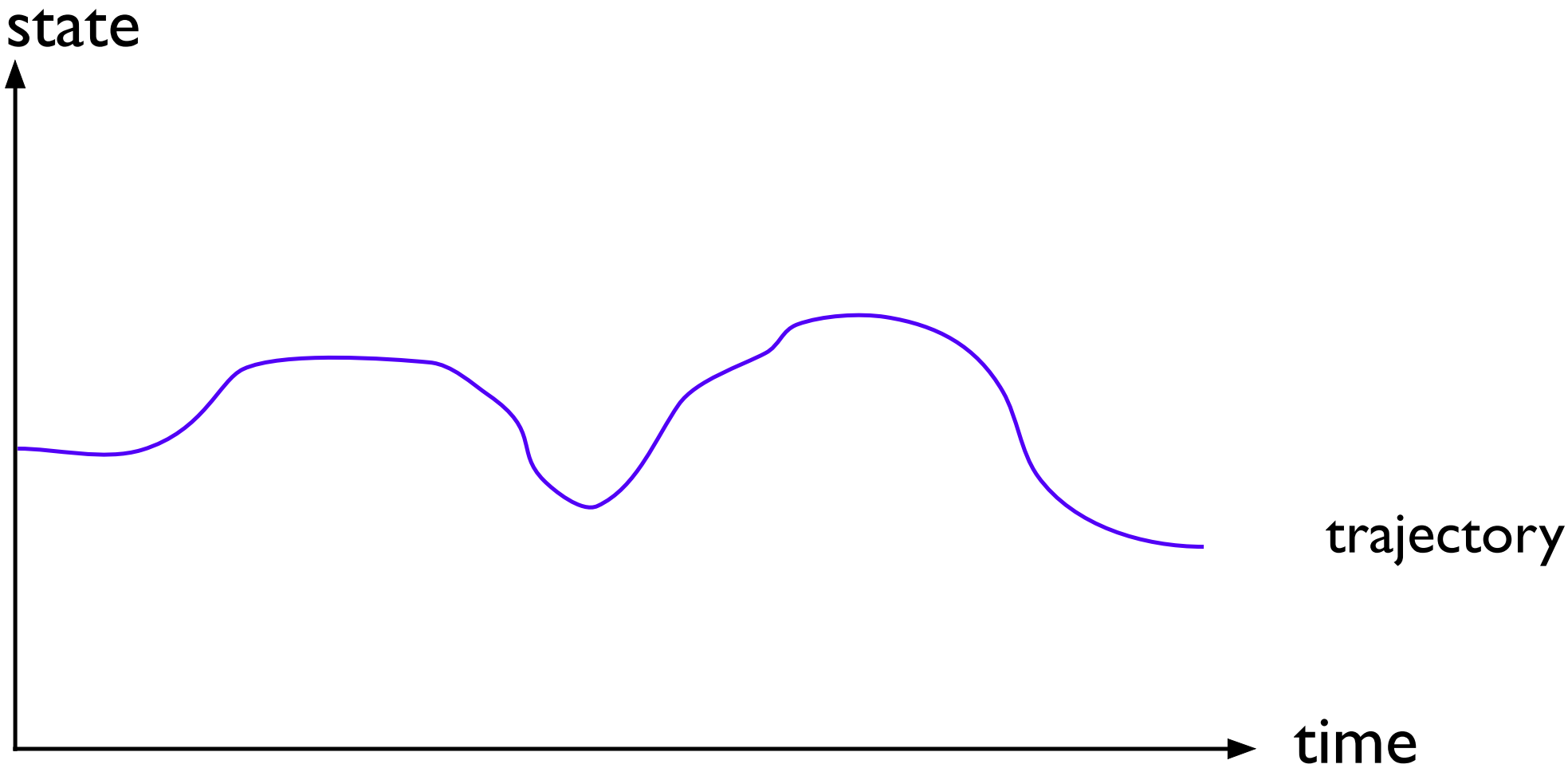


Graphical representation of the execution (2)



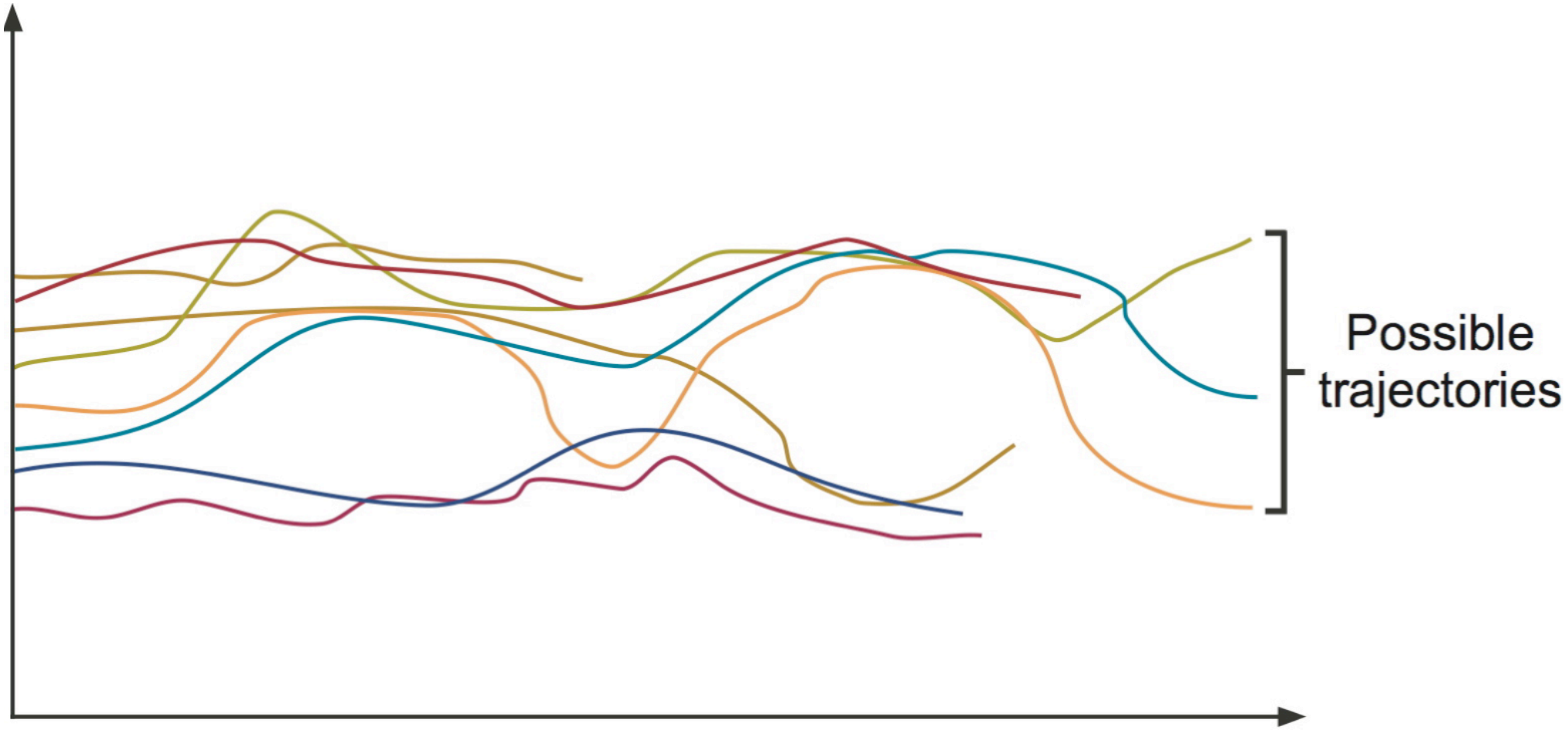
Semantics

Formalize what it means to run a program



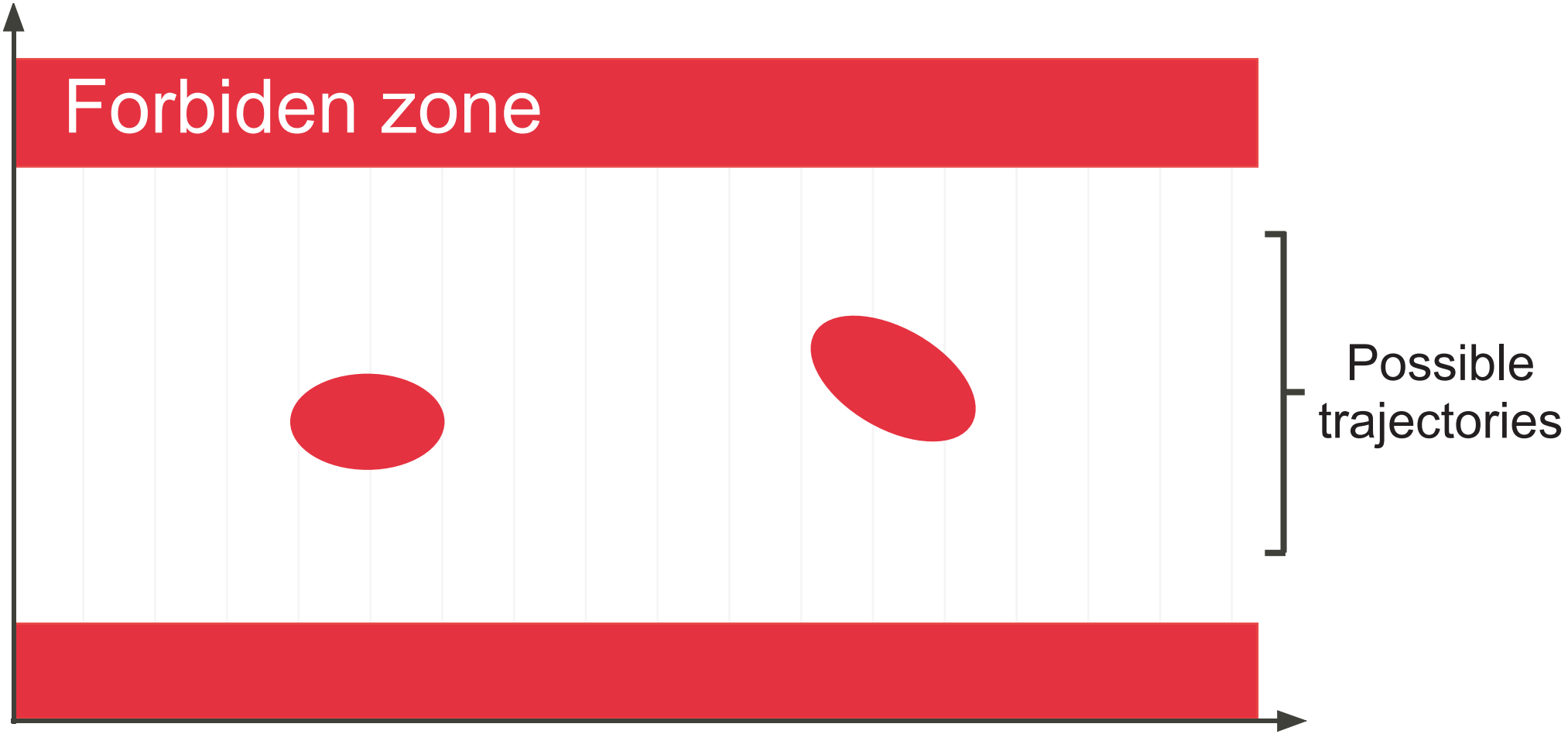
Properties (Collecting semantics)

*Formalize what you are interested to **know** about program behaviors*



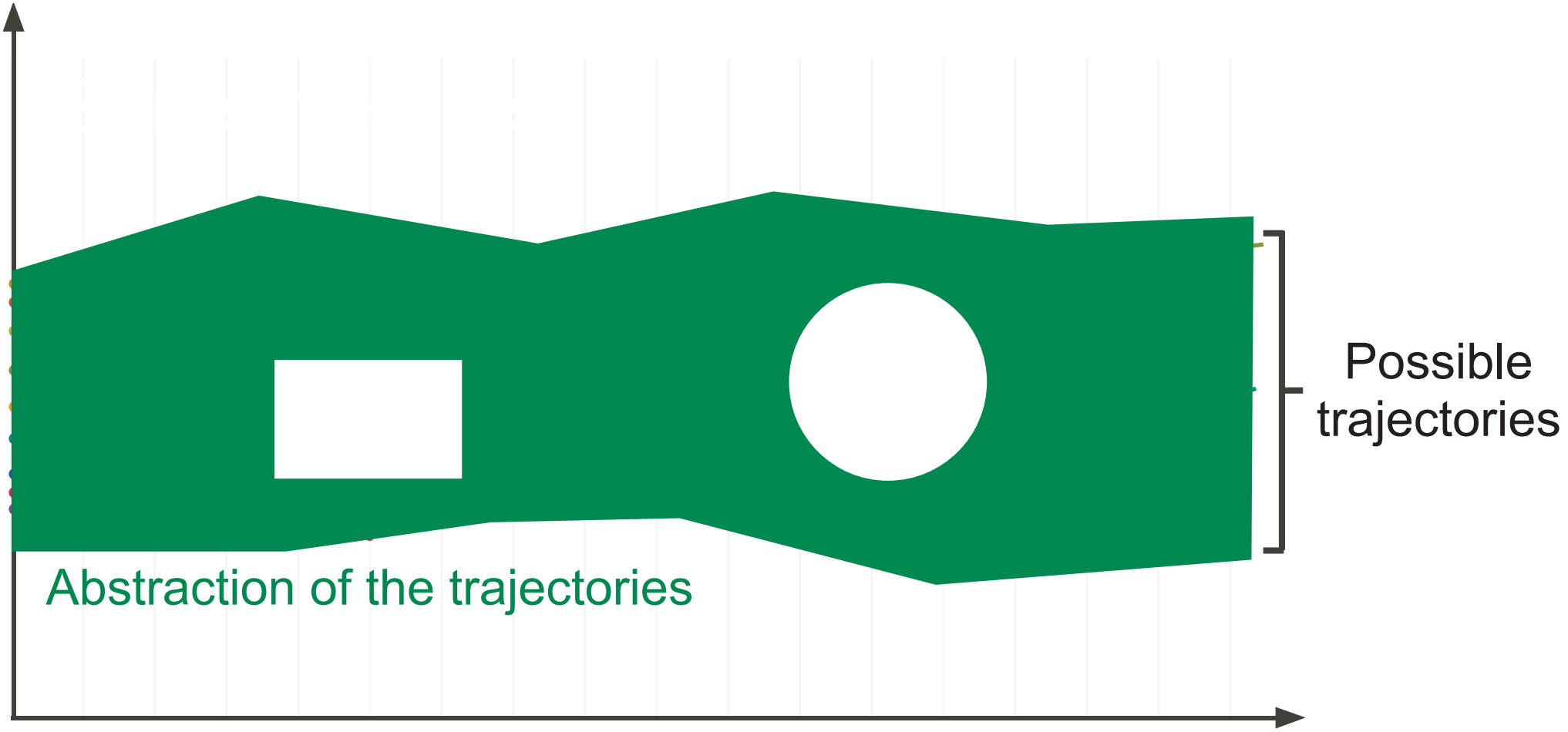
Specification

*Formalize what you are interested to **prove** about program behaviors*



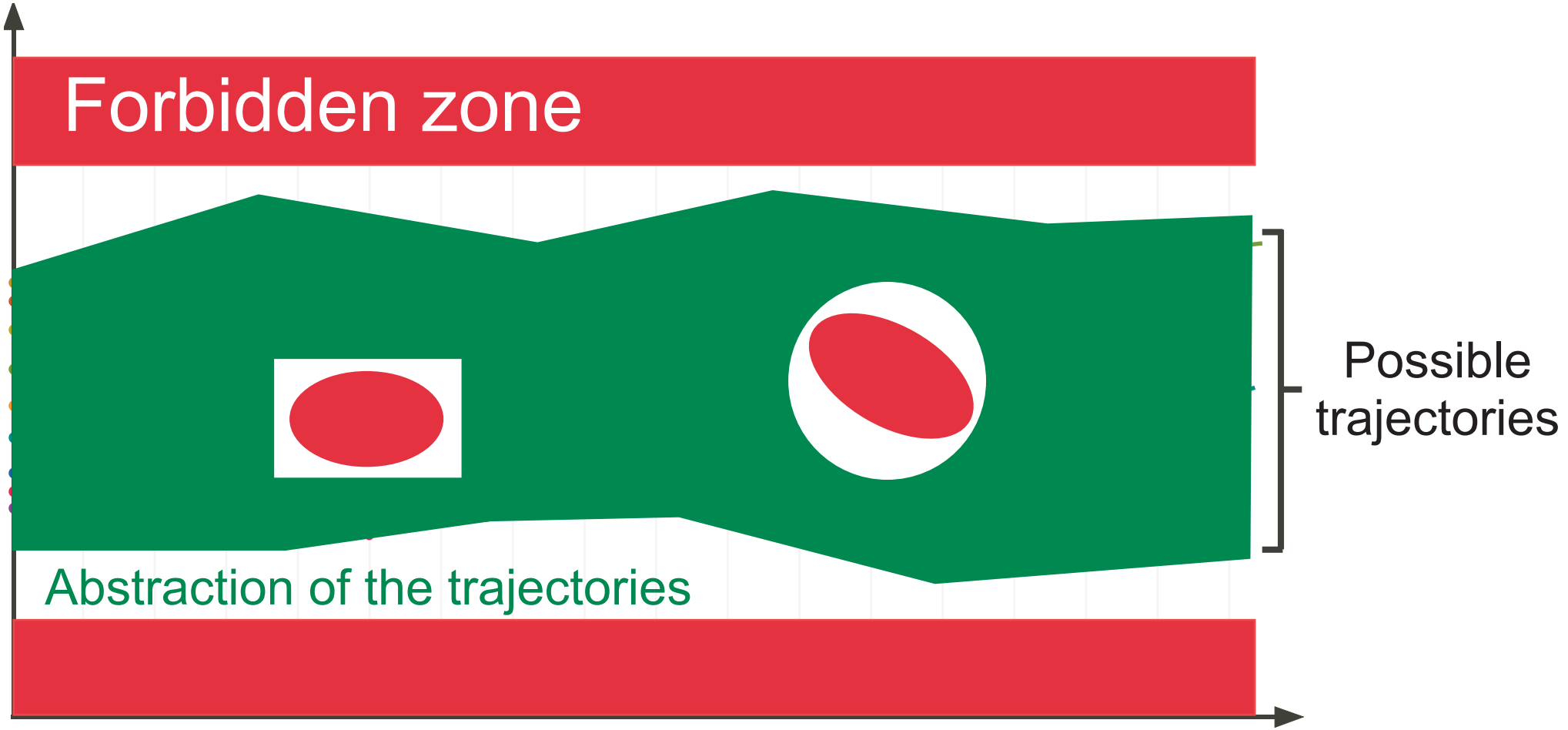
Abstraction

Abstract *away all information on program behaviors irrelevant to the proof*



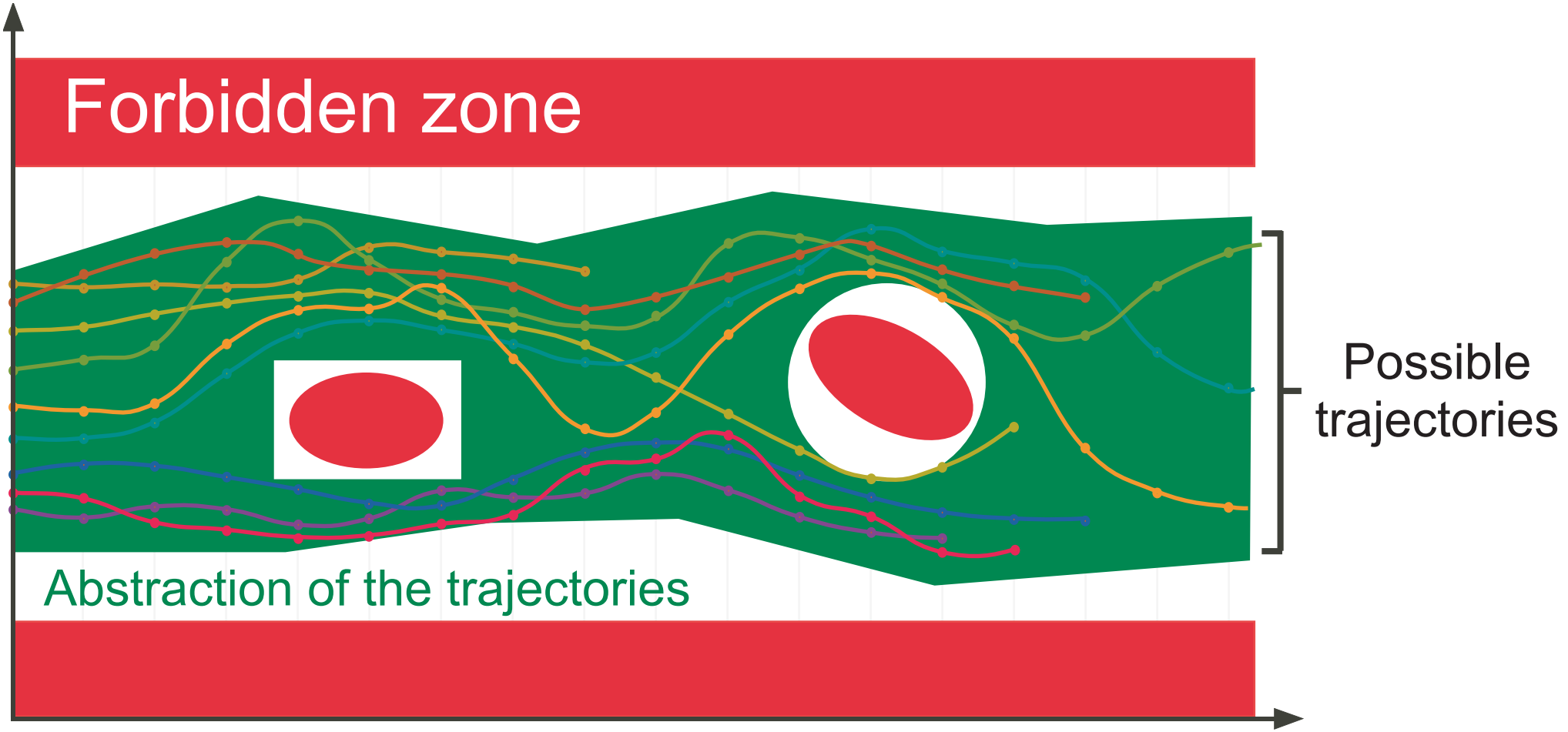
Verification

The proof is fully ***automatic***



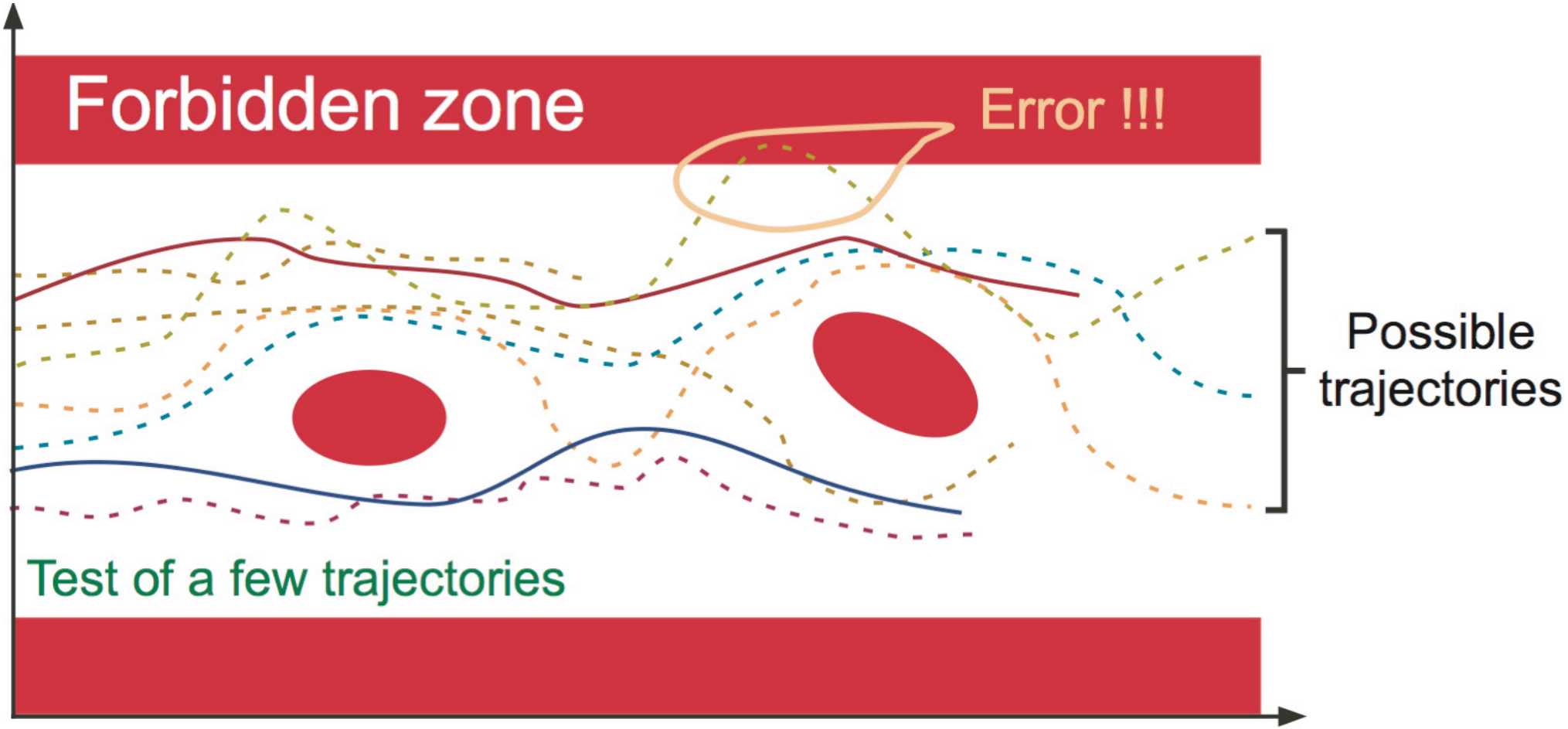
Soundness

*Never forget any possible case so the **abstract proof is correct in the concrete***



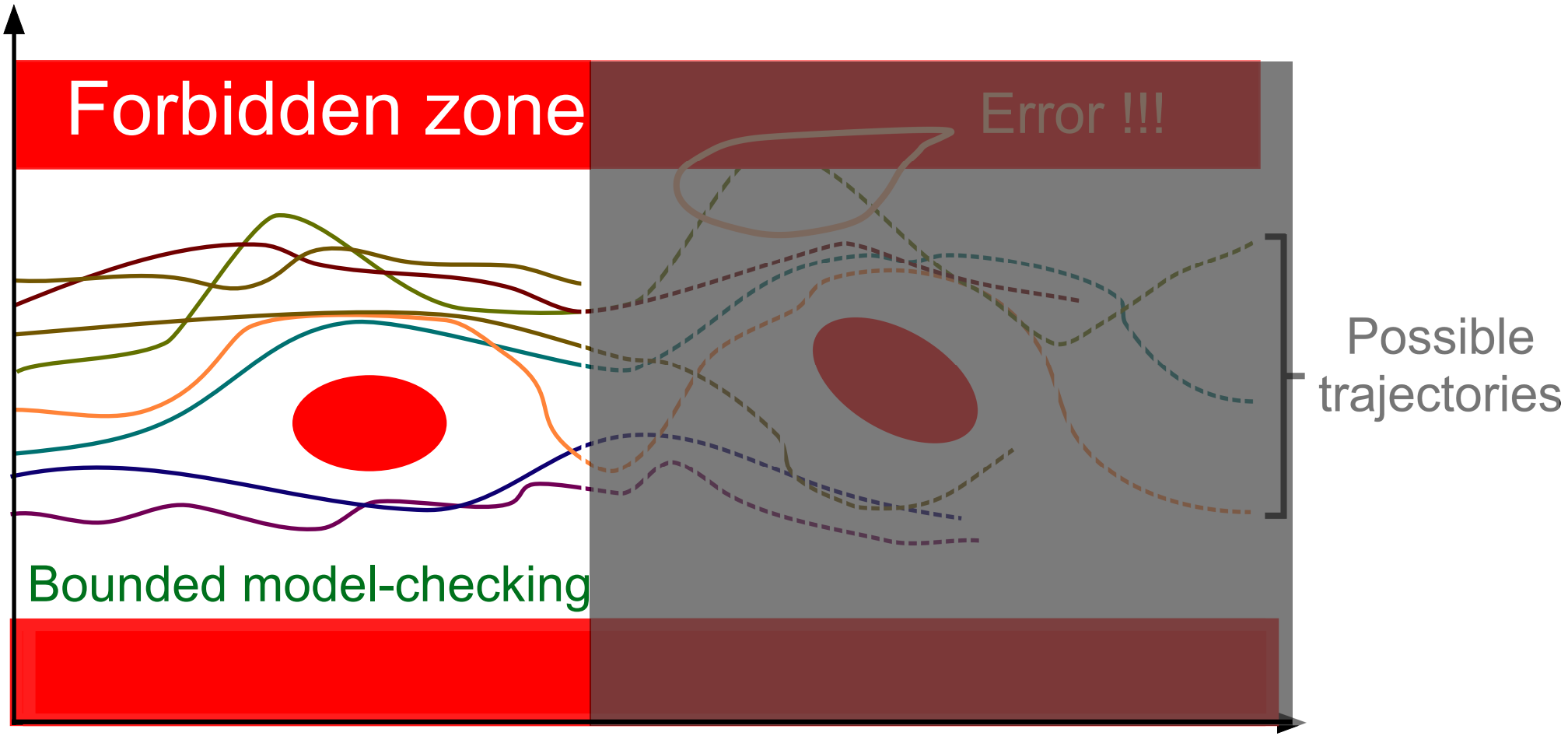
Unsound methods: testing

Try a few cases



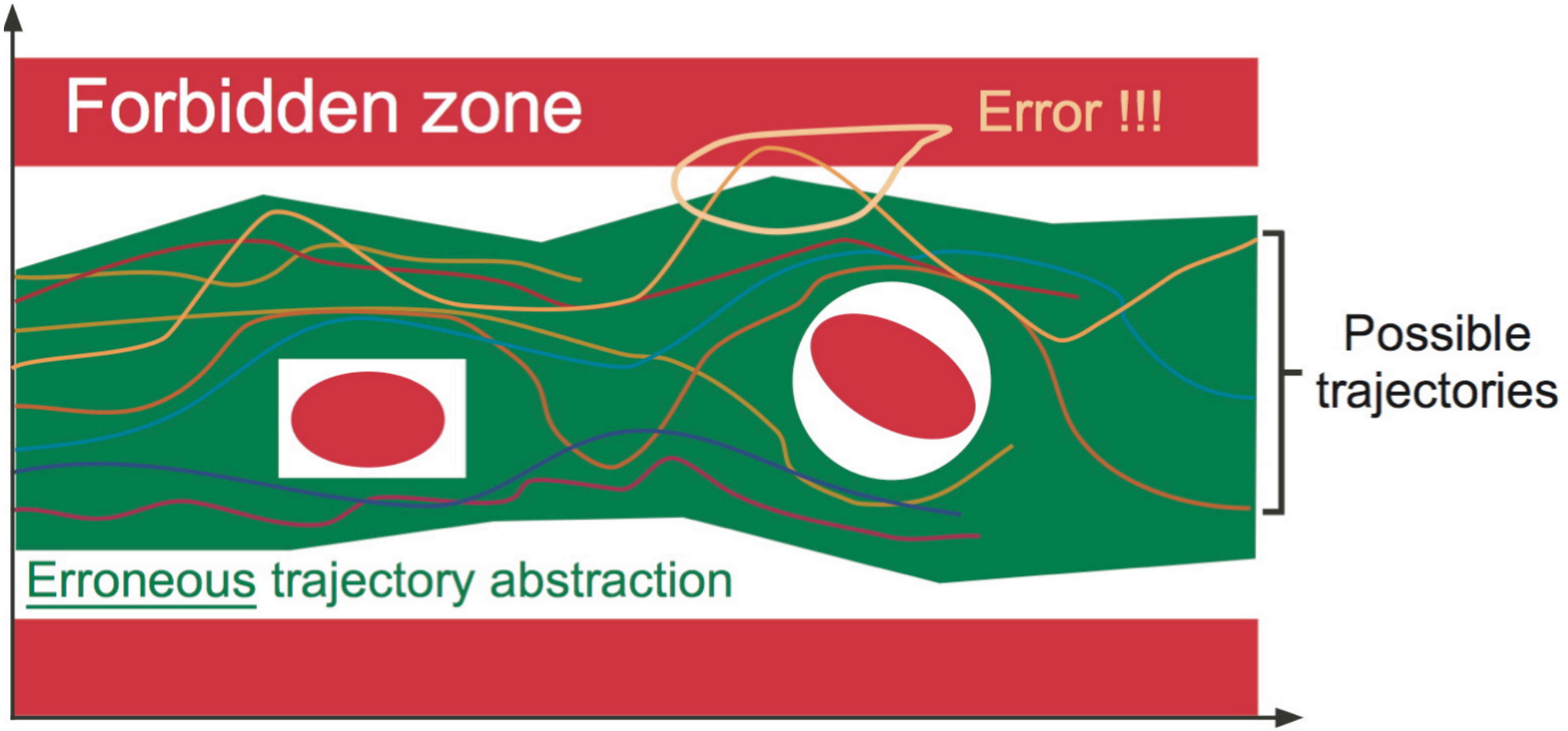
Unsound methods: bounded model checking

Simulate the beginning of all executions (so called bounded model-checking)



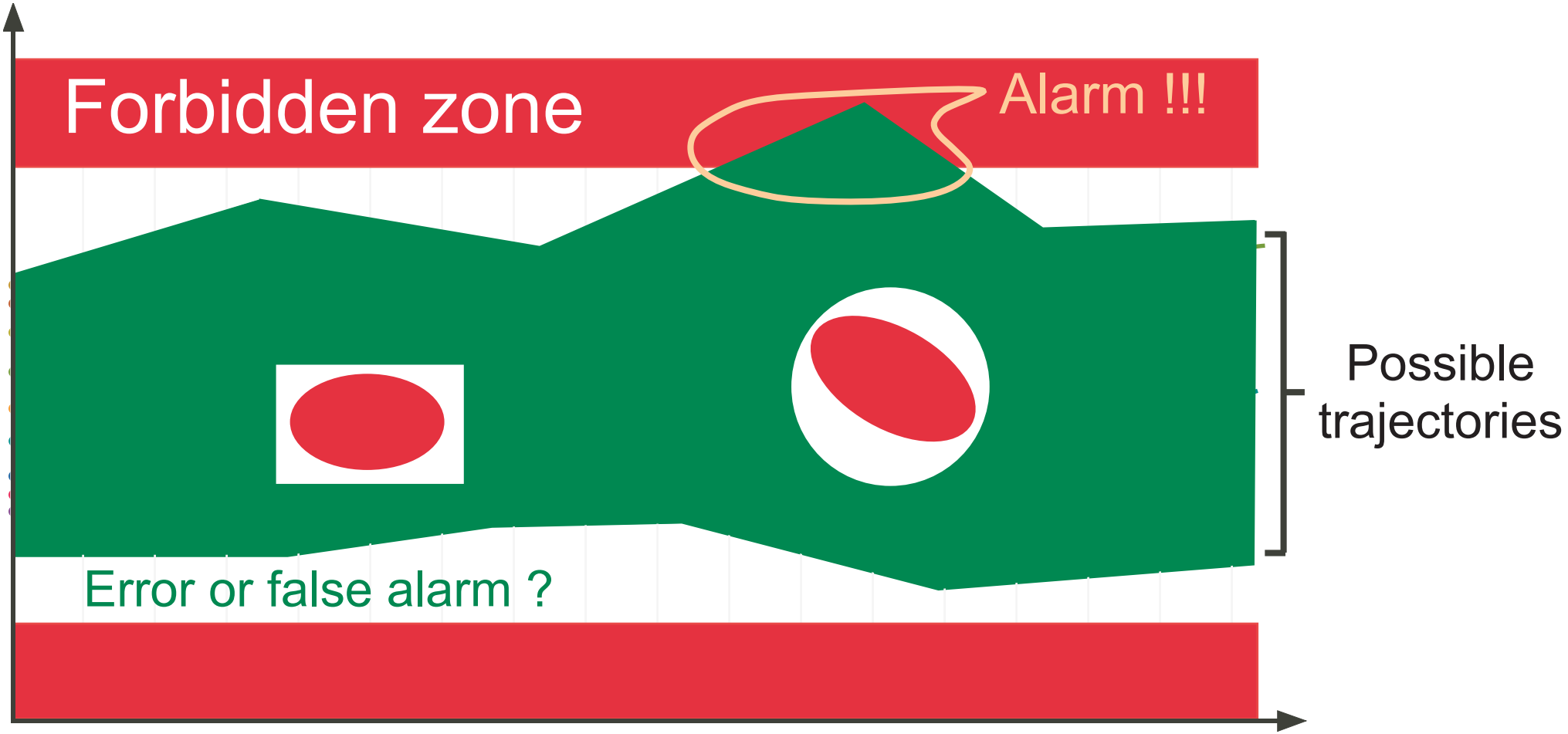
Unsound methods: soundiness

*Many static analysis tools are **unsound** (e.g. Coverity, etc.) so inconclusive*



Alarms

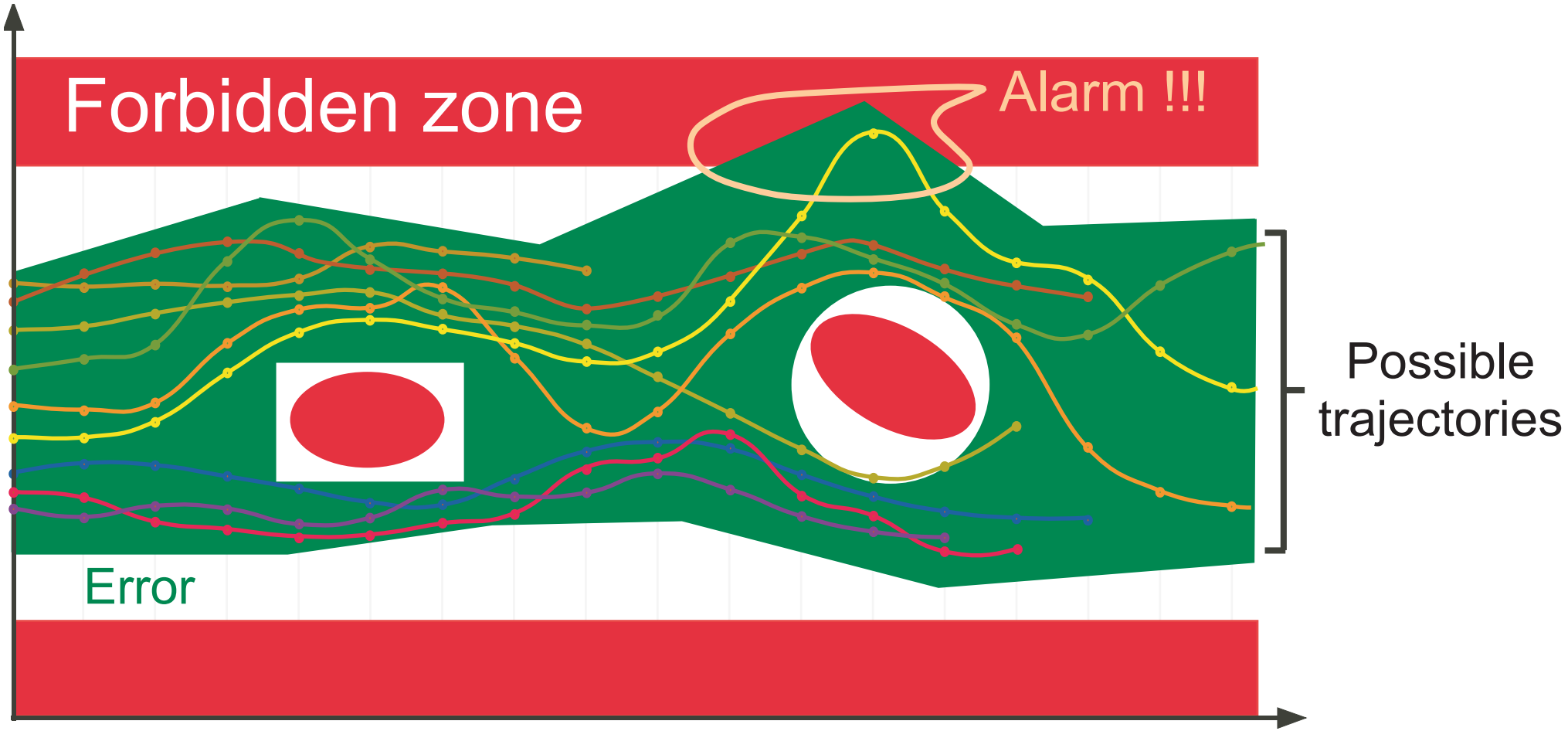
When abstract proofs may fail while concrete proofs would succeed



By soundness an alarm must be raised for this over-approximation!

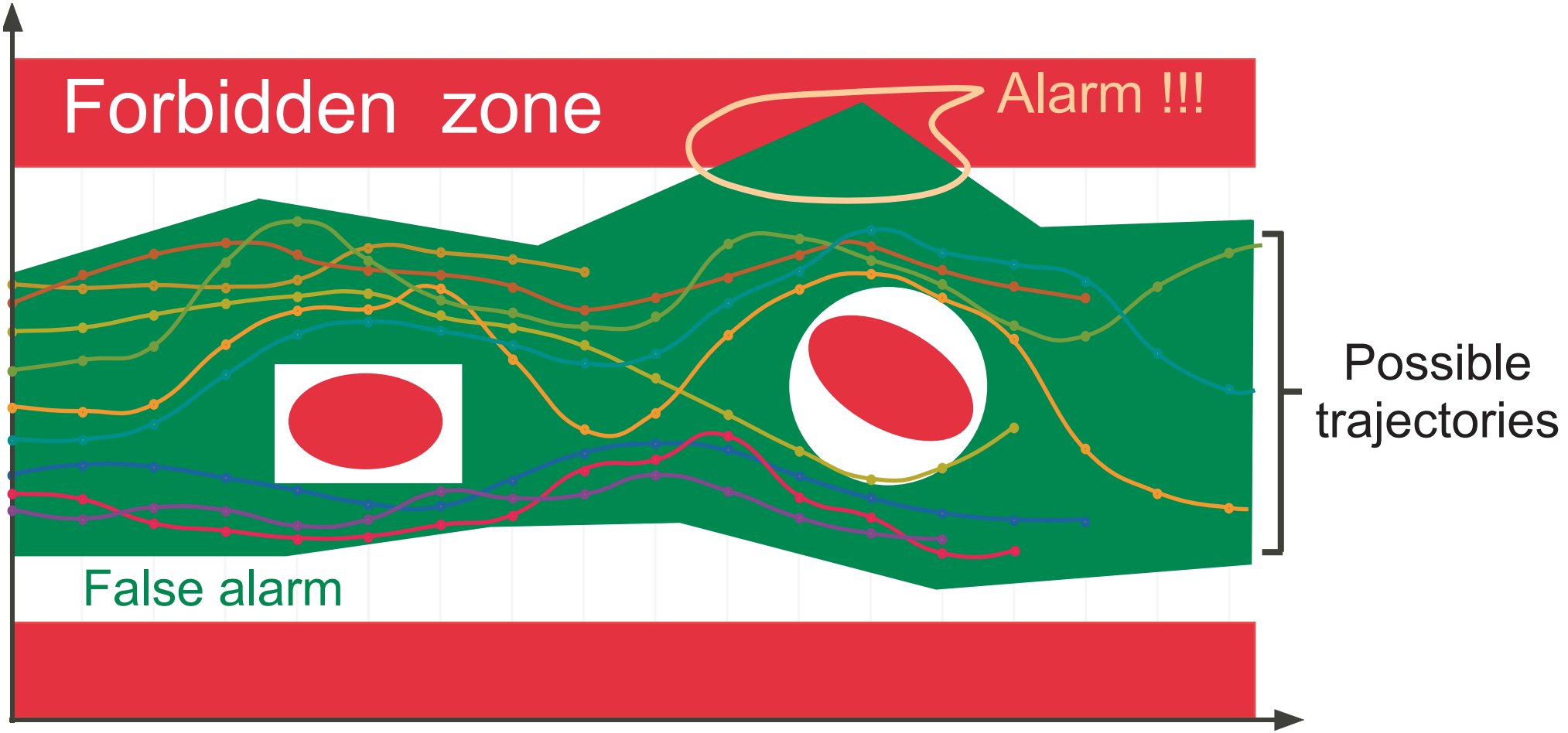
True alarm

The abstract alarm may correspond to a concrete error



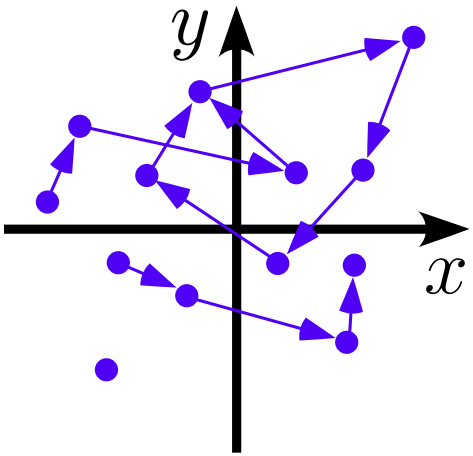
False alarm

The abstract alarm may correspond to no concrete error (false negative)

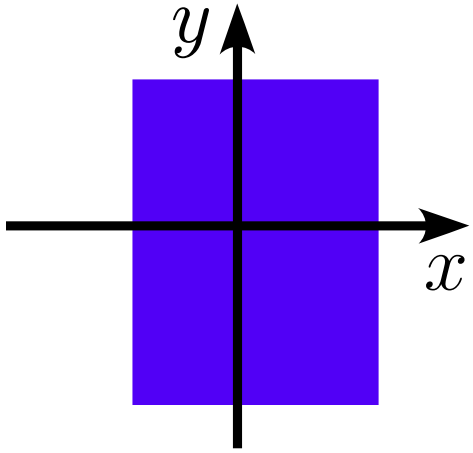


What to do in presence of false alarms

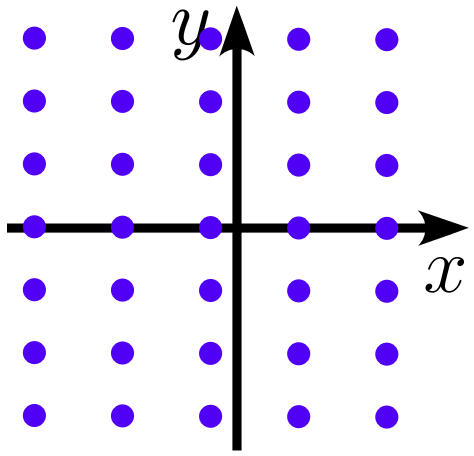
- False alarms are ultimately unavoidable (Gödel's incompleteness)
- Consider finite cases or decidable cases only (model-checking, *does not scale*)
- Ask for human help by providing information on the program behavior (theorem provers, SMT solvers), *program specific and labor costly*
- Have specialists refine the abstract interpretation (e.g. Astrée, http://www.absint.com/astree/index.htm), *shared cost*



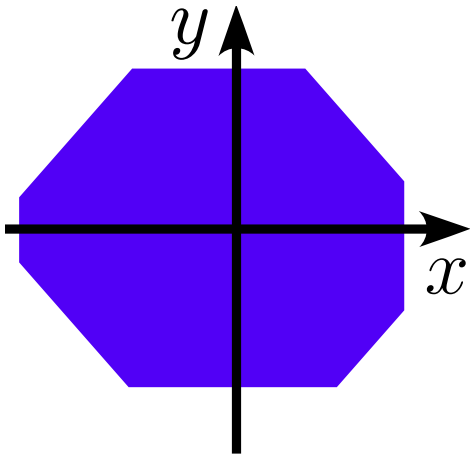
Collecting semantics:
partial traces



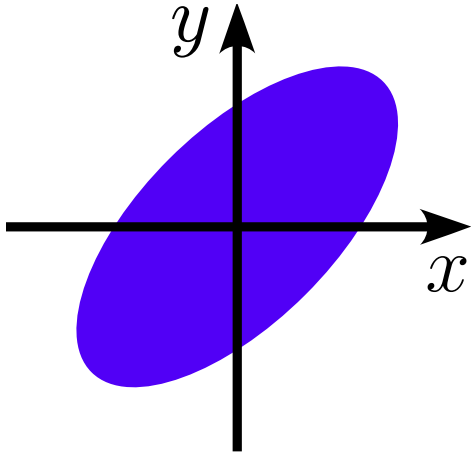
Intervals:
$\mathtt{x} \in [a, b]$



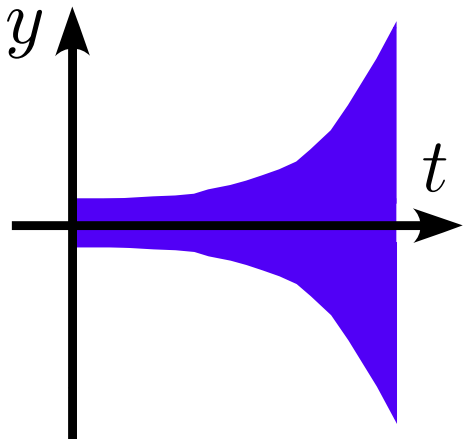
Simple congruences:
$\mathtt{x} \equiv a[b]$



Octagons:
$\pm\mathtt{x} \pm \mathtt{y} \leqslant a$



Ellipses:
$\mathtt{x}^2 + b\mathtt{y}^2 - a\mathtt{x}\mathtt{y} \leqslant d$



Exponentials:
$-a^{bt} \leqslant \mathtt{y}(t) \leqslant a^{bt}$

The very first static analysis

Brahmagupta

Brahmagupta (Sanskrit: ब्रह्मगुप्त; (598–c.670 CE) was an Indian mathematician and astronomer who wrote two important works on Mathematics and Astronomy: the *Brāhmasphuṭasiddhānta* (Extensive Treatise of Brahma) (628), a theoretical treatise, and the *Khaṇḍakhādyaka*, a more practical text.

Brahmagupta

Born	598 CE
Died	c.670 CE
Fields	Mathematics, Astronomy
Known for	Zero, modern Number system

The rule of signs by Brahmagupta (628)

18.30. [The sum] of two positives is positives, of two negatives negative;

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- The abstraction is that you do not (always) need to known the absolute value of the arguments to know the sign of the result;

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- Sometimes imprecise (don't know the sign of the sum of a positive and a negative)

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- Useful in practice (if you know what to do when you don't know the sign)

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- The abstraction is that you do not (always) need to known the absolute value of the arguments to know the sign of the result;
- Sometimes imprecise (don't know the sign of the sum of a positive and a negative)
- Useful in practice (if you know what to do when you don't know the sign)
- e.g. in compilation: do not optimize (a division by 2 into a shift when positive(*))

(*) Unless processor uses 2's complement and can shift the sign.

The rule of signs by Brahmagupta (628)

18.30. [The sum] of two positives is positives, of two negatives negative; [...]

18.32. A negative minus zero is negative, a positive [minus zero] positive; zero [minus zero] is zero. When a positive is to be subtracted from a negative or a negative from a positive, then it is to be added.

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18.33. The product of a negative and a positive is negative, of two negatives positive, and of positives positive; the product of zero and a negative, of zero and a positive, or of two zeros is zero.

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18.33. The product of a negative and a positive is negative, of two negatives positive, and of positives positive; the product of zero and a negative, of zero and a positive, or of two zeros is zero.

18.34. A positive divided by a positive or a negative divided by a negative is positive; a zero divided by a zero is zero; a positive divided by a negative is negative; a negative divided by a positive is [also] negative.

wrong

The rule of signs by Michel Sintzoff (1972)

For example, a×a+b×b yields the value 25
when a is 3 and b is -4, and when + and × are
the arithmetic multiplication and addition.
But a×a+b×b yields always the object "pos" when
a and b are the objects "pos" or "neg", and when
the valuation is defined as follows :

pos+pos=pos	pos×pos=pos
pos+neg=pos,neg	pos×neg=neg
neg+pos=pos,neg	neg×pos=neg
neg+neg=neg	neg×neg=pos
V(p+q)=V(p)+V(q)	V(p×q)=V(p)×V(q)

V(0)=V(1)=...=pos
V(-1)=V(-2)=...=neg
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V(a×a)=pos×pos,neg×neg	V(b×b)=pos×pos,neg×neg
=pos,pos=pos	=pos,pos=pos

V(a×a+b×b)=V(a×a)+V(b×b)=pos+pos=pos
This valuation proves that the result of
a×a+b×b is always positive and hence allows to
compute its square root without any preliminary
dynamic test on its sign. On the other hand, the

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V(0)=V(1)=...=pos	
V(-1)=V(-2)=...=neg	

wrong

$0 \in pos \times -1 \in neg$
$= 0 \notin neg$

The valuation of a×a+b×b yields "pos" by the
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V(a×a)=pos×pos,neg×neg	V(b×b)=pos×pos,neg×neg
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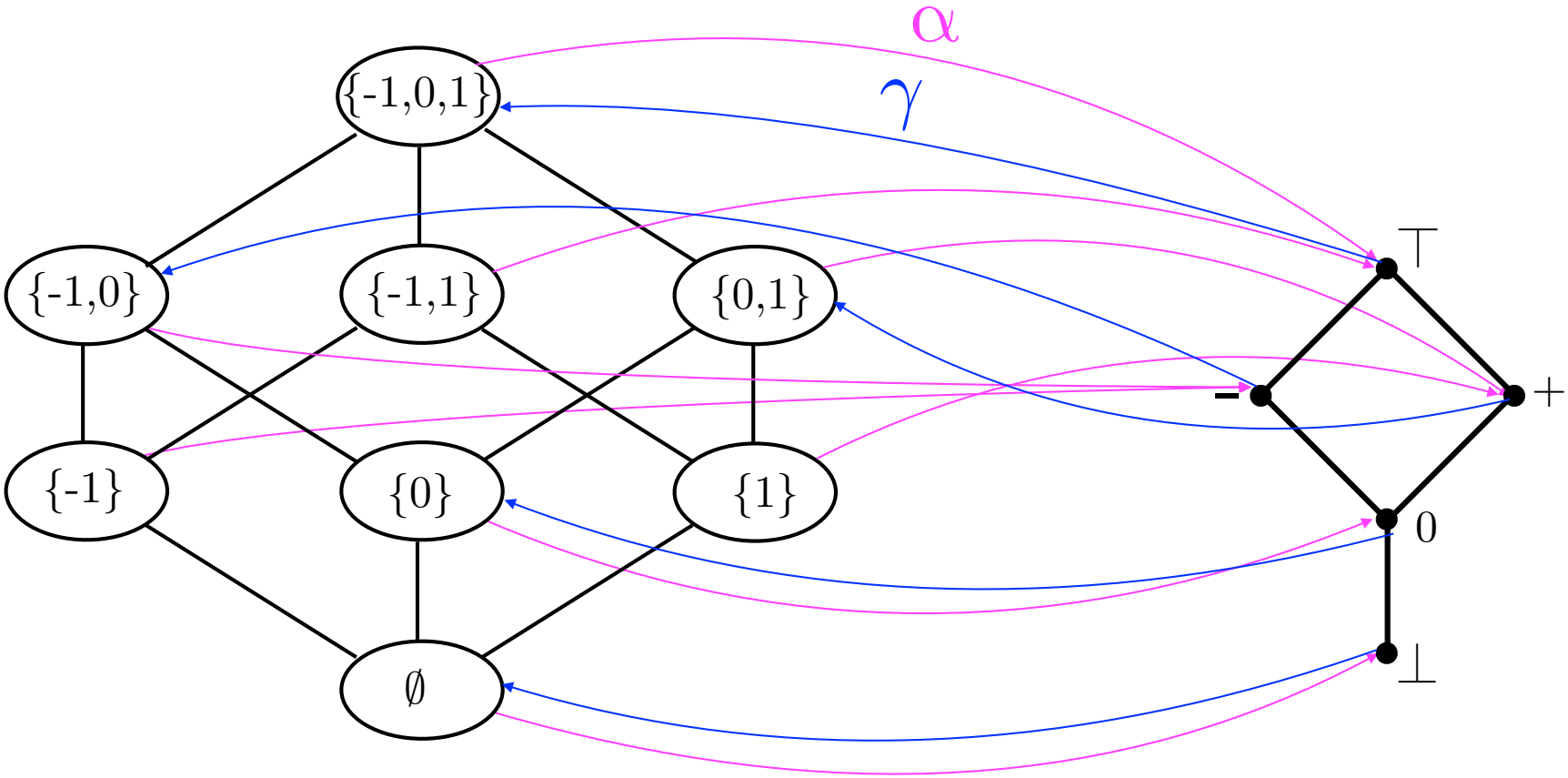
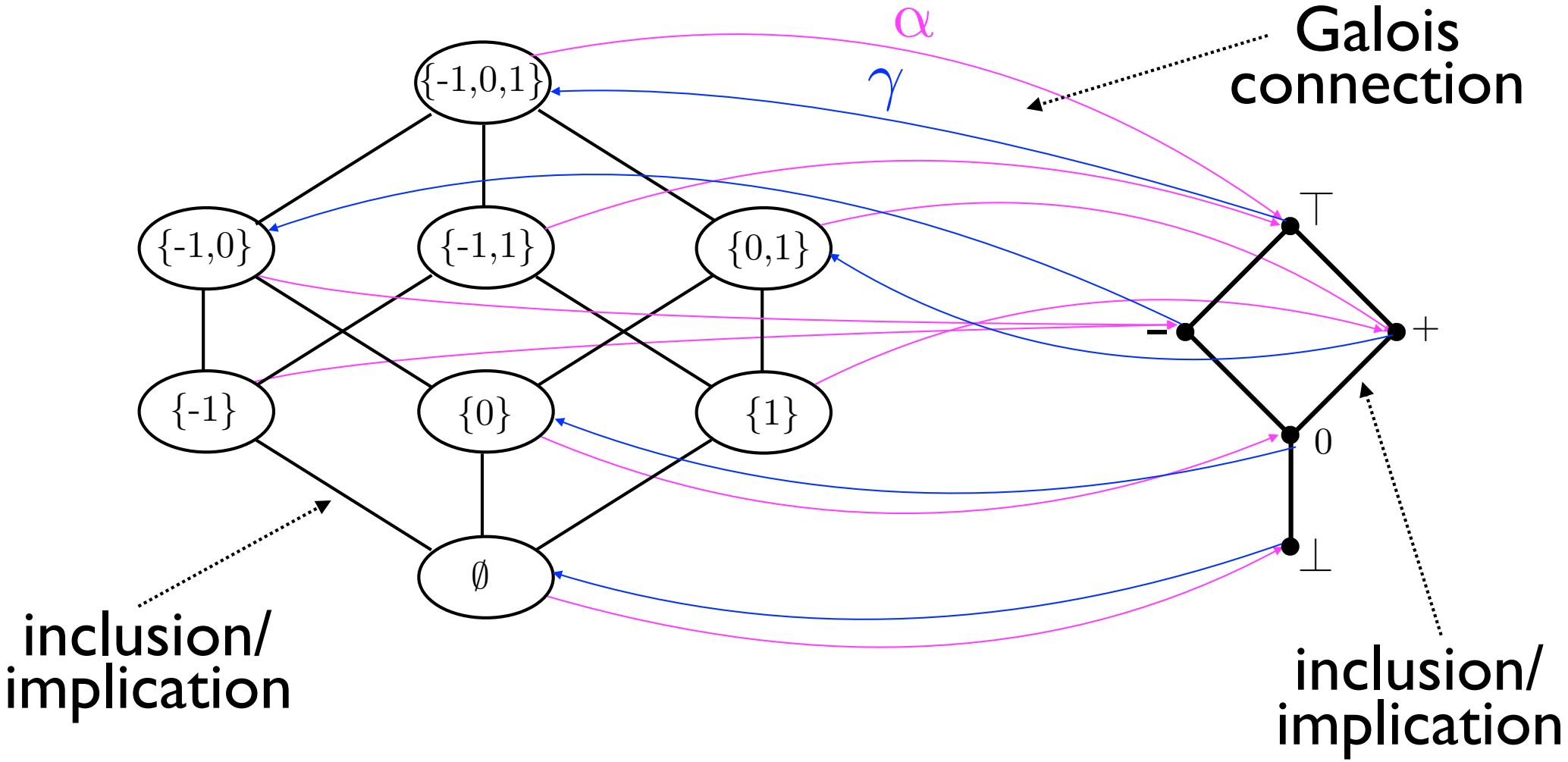
The rule of signs Cousot & Cousot (1979)



The rule of signs Cousot & Cousot (1979)



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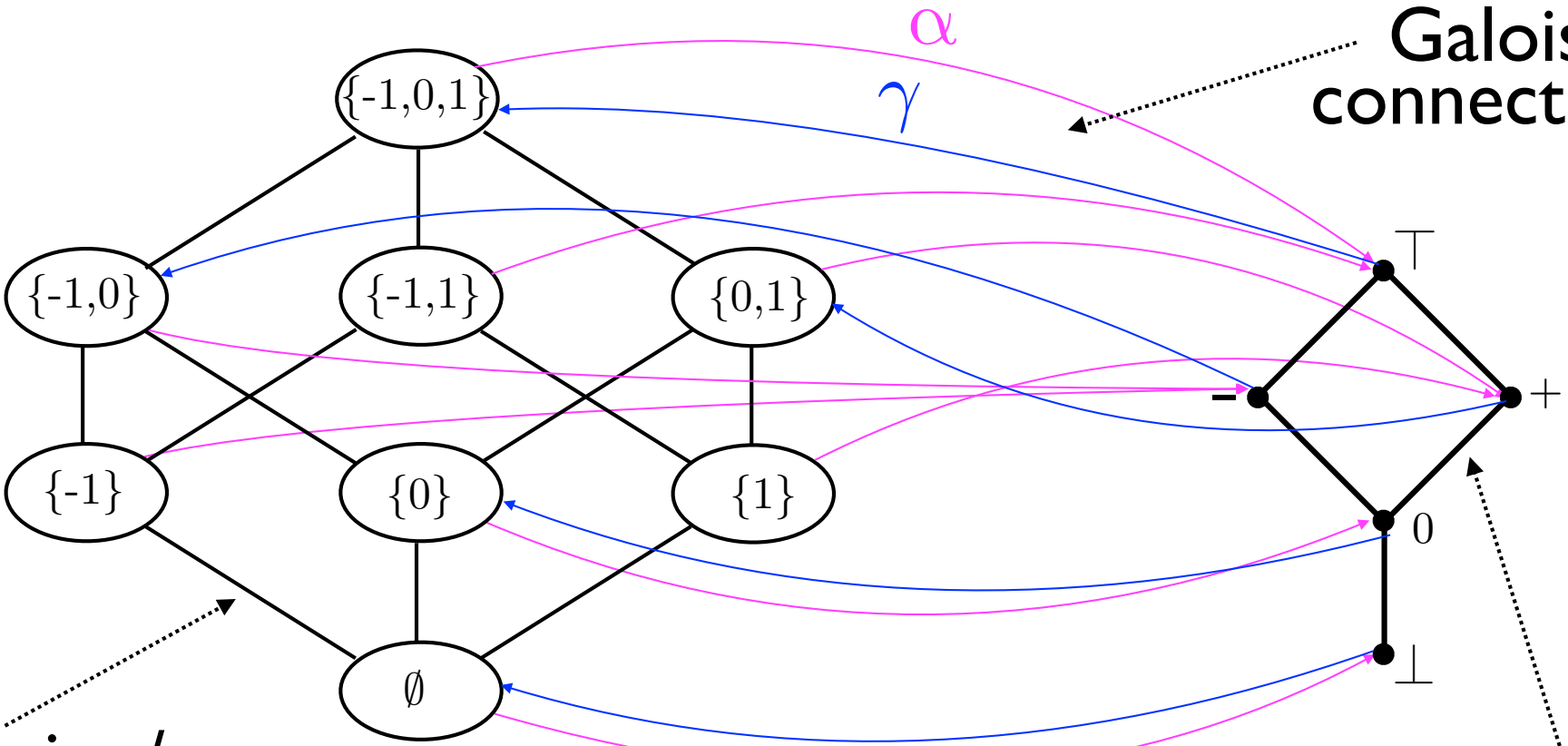


Galois connection

inclusion/ implication

inclusion/ implication

calculational design method

$$+ + + \quad = \quad +$$

$$\gamma \quad \gamma \quad \alpha$$

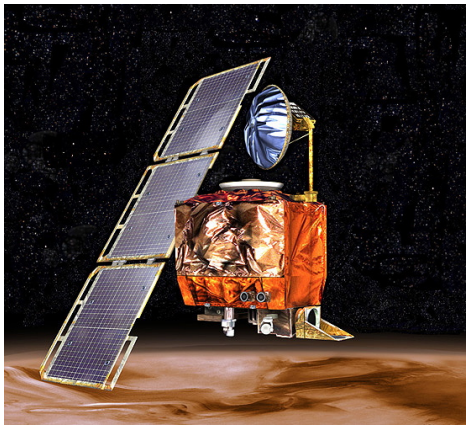
$$\{0,1\} + \{0,1\} = \{0,1,2[2]\} = \{0,1\}$$

Application of abstract interpretation to static analysis

All computer scientists have experienced bugs

Ariane 5.01 failure
(overflow)

Patriot failure
(float rounding)



Mars orbiter loss
(unit error)

Heartbleed
(buffer overrun)

- Checking the presence of bugs by debugging is great
- Proving their absence by static analysis is even better!

Static analysis

- Check program properties (automatically, using the program text only, without running the program)
- Difficulties:
  - Undecidability / complexity:
    - Precision
    - Scalability
  - Soundness (correctness)
  - Induction: widening/narrowing

Fixpoint

Fixpoint equation

$\{y \geqslant 0\}$ ← hypothesis

```
x = y
```

$\{I(x,y)\}$ ← loop invariant

```
while (x > 0) {
  x = x - 1;
}
```

Floyd-Naur-Hoare verification conditions:

$$(y \geqslant 0 \wedge x = y) \Longrightarrow I(x,y) \qquad \textit{initialisation}$$

$$(I(x,y) \wedge x > 0 \wedge x' = x - 1) \Longrightarrow I(x',y) \qquad \textit{iteration}$$

Equivalent fixpoint equation:

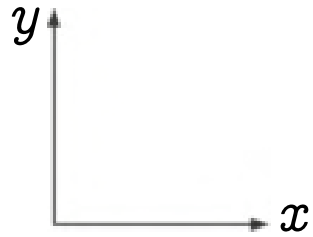
$$I(x,y) \;=\; x \geqslant 0 \wedge (x = y \vee I(x+1,y)) \qquad (\textit{i.e. } I = F(I)\,^{(5)})$$

(5) We look for the most precise invariant I, implying all others, that is $\mathsf{lfp}^{\Longrightarrow} F$.

Iterates

$$\text{Iterates } I = \lim_{n \to \infty} F^n(\text{false})$$

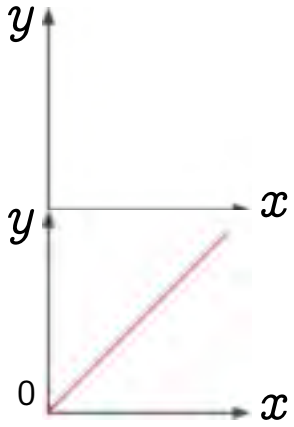
$$I^0(x, y) = \text{false}$$



Iterates

$$\text{Iterates } I = \lim_{n \to \infty} F^n(\text{false})$$

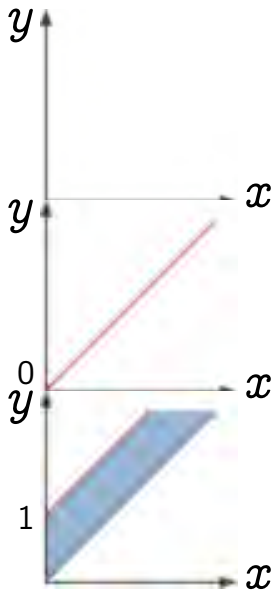
$$\begin{aligned} I^0(x,y) &= \text{false} \\ I^1(x,y) &= x \geqslant 0 \wedge (x = y \vee I^0(x+1,y)) \\ &= 0 \leqslant x = y \end{aligned}$$



Iterates

$$\text{Iterates } I = \lim_{n \to \infty} F^n(\text{false})$$

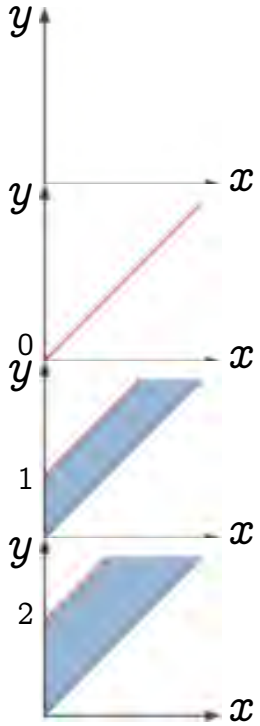
$$\begin{aligned} I^0(x,y) &= \text{false} \\ I^1(x,y) &= x \geqslant 0 \wedge (x = y \vee I^0(x+1,y)) \\ &= 0 \leqslant x = y \\ I^2(x,y) &= x \geqslant 0 \wedge (x = y \vee I^1(x+1,y)) \\ &= 0 \leqslant x \leqslant y \leqslant x+1 \end{aligned}$$



Iterates

$$\text{Iterates } I = \lim_{n \to \infty} F^n(\text{false})$$

$$\begin{aligned}
I^0(x,y) &= \text{false} \\
I^1(x,y) &= x \geqslant 0 \wedge (x = y \vee I^0(x+1,y)) \\
&= 0 \leqslant x = y \\
I^2(x,y) &= x \geqslant 0 \wedge (x = y \vee I^1(x+1,y)) \\
&= 0 \leqslant x \leqslant y \leqslant x+1 \\
I^3(x,y) &= x \geqslant 0 \wedge (x = y \vee I^2(x+1,y)) \\
&= 0 \leqslant x \leqslant y \leqslant x+2
\end{aligned}$$



Convergence acceleration: widening

Accelerated Iterates $I = \lim_{n \to \infty} F^n(\text{false})$

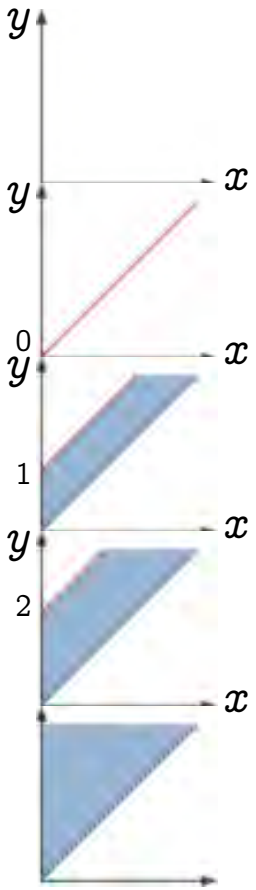
$$I^0(x,y) = \text{false}$$

$$I^1(x,y) = x \geqslant 0 \wedge (x = y \vee I^0(x+1,y)) = 0 \leqslant x = y$$

$$I^2(x,y) = x \geqslant 0 \wedge (x = y \vee I^1(x+1,y)) = 0 \leqslant x \leqslant y \leqslant x+1$$

$$I^3(x,y) = x \geqslant 0 \wedge (x = y \vee I^2(x+1,y)) = 0 \leqslant x \leqslant y \leqslant x+2$$

$$I^4(x,y) = I^2(x,y) \triangledown I^3(x,y) \leftarrow \text{widening} = 0 \leqslant x \leqslant y$$



Fixed point

Accelerated Iterates $I = \lim_{n \to \infty} F^n(\text{false})$

$$I^0(x,y) = \text{false}$$

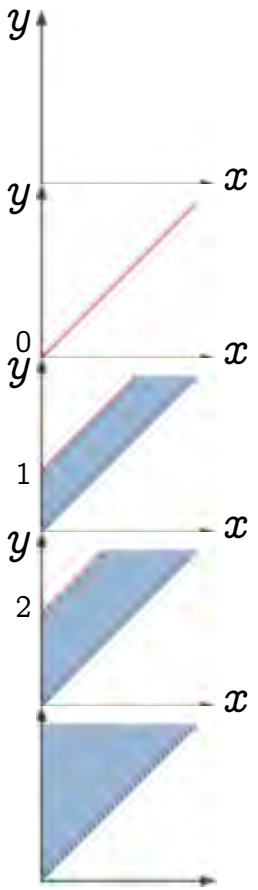
$$\begin{aligned} I^1(x,y) &= x \geqslant 0 \wedge (x = y \vee I^0(x+1,y)) \\ &= 0 \leqslant x = y \end{aligned}$$

$$\begin{aligned} I^2(x,y) &= x \geqslant 0 \wedge (x = y \vee I^1(x+1,y)) \\ &= 0 \leqslant x \leqslant y \leqslant x+1 \end{aligned}$$

$$\begin{aligned} I^3(x,y) &= x \geqslant 0 \wedge (x = y \vee I^2(x+1,y)) \\ &= 0 \leqslant x \leqslant y \leqslant x+2 \end{aligned}$$

$$\begin{aligned} I^4(x,y) &= I^2(x,y) \mathbin{\triangledown} I^3(x,y) \leftarrow \text{widening} \\ &= 0 \leqslant x \leqslant y \end{aligned}$$

$$\begin{aligned} I^5(x,y) &= x \geqslant 0 \wedge (x = y \vee I^4(x+1,y)) \\ &= I^4(x,y) \quad \text{fixed point!} \end{aligned}$$



Octagons

Accelerated Iterates $I = \lim_{n \to \infty} F^n(\text{false})$

$$I^0(x,y) = \text{false}$$

$$\begin{aligned} I^1(x,y) &= x \geqslant 0 \wedge (x = y \vee I^0(x+1,y)) \\ &= 0 \leqslant x = y \end{aligned}$$

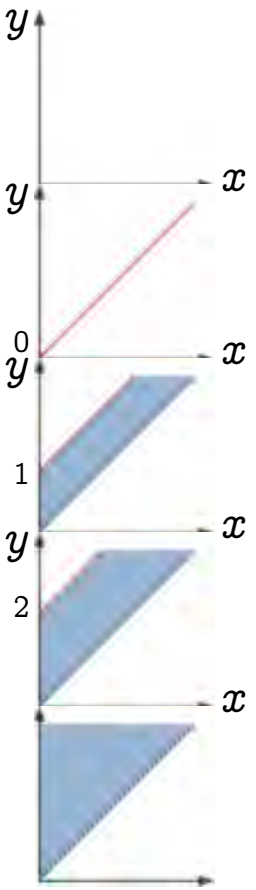
$$\begin{aligned} I^2(x,y) &= x \geqslant 0 \wedge (x = y \vee I^1(x+1,y)) \\ &= 0 \leqslant x \leqslant y \leqslant x+1 \end{aligned}$$

$$\begin{aligned} I^3(x,y) &= x \geqslant 0 \wedge (x = y \vee I^2(x+1,y)) \\ &= 0 \leqslant x \leqslant y \leqslant x+2 \end{aligned}$$

$$\begin{aligned} I^4(x,y) &= I^2(x,y) \mathbin{\triangledown} I^3(x,y) \leftarrow \text{widening} \\ &= 0 \leqslant x \leqslant y \end{aligned}$$

$$\begin{aligned} I^5(x,y) &= x \geqslant 0 \wedge (x = y \vee I^4(x+1,y)) \\ &= I^4(x,y) \quad \text{fixed point!} \end{aligned}$$

The invariants are computer representable with octagons!



Industrialisation: Development in cooperation with Airbus France

– Automatic proofs of absence of runtime errors in Electric Flight Control Software:
  – A340/600: 132.000 lines of C, 40mn on a PC 2.8 GHz, 300 Mb (Nov. 2003)
  – A380: 1.000.000 lines of C, 34h, 8 Gb (Nov. 2005)

  no false alarm, **World premières !**

– Automatic proofs of absence of runtime errors in the ATV software [2]:
  – C version of the automatic docking software: 102.000 lines of C, 23s on a Quad-Core AMD Opteron™ processor, 16 Gb (Apr. 2008)

[2] the Jules Vernes Automated Transfer Vehicle (ATV) enabling ESA to transport payloads to the International Space Station.

Application of abstract interpretation to program proof methods

Maximal execution trace

```
#include <stdio.h>
int main() {
          int x,y;
          printf("Enter an integer: ");
          scanf("%d",&x); y = x;
/* 1: */  while (x != 0) {
             printf("x = %d, y = %d\n",x,y);
/* 2: */     x = x - 1;
/* 3: */     y = y + 2;
          }
/* 4: */  printf("x = %d, y = %d\n",x,y); }
```

```
Enter an integer: 3
x = 3, y = 3
x = 2, y = 5
x = 1, y = 7
x = 0, y = 9
```

```
Enter an integer: -1
x = -1, y = -1
x = -2, y = 1
x = -3, y = 3
x = -4, y = 5
…
x = -738245, y = 1476487
…
```

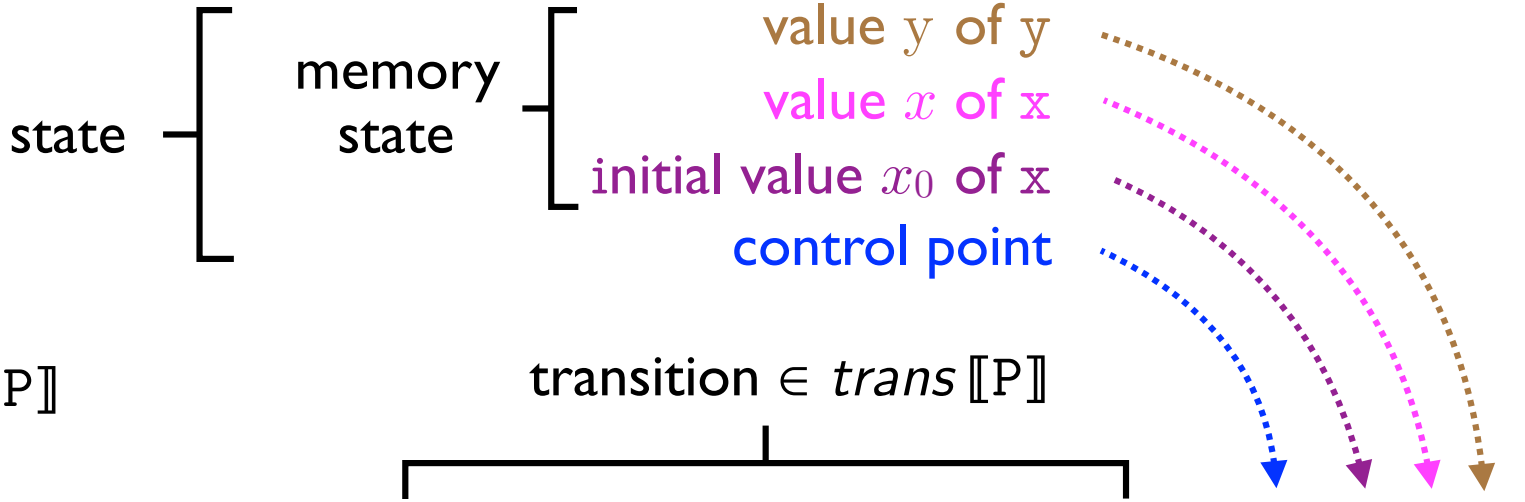
$$\langle 1:,3,3,3\rangle \longrightarrow \langle 2:,3,3,3\rangle \longrightarrow \langle 3:,3,2,3\rangle \longrightarrow \langle 1:,3,2,5\rangle \longrightarrow \langle 2:,3,2,5\rangle \longrightarrow \langle 3:,3,1,5\rangle \longrightarrow \langle 1:,3,1,7\rangle \longrightarrow \langle 2:,3,1,7\rangle \longrightarrow \langle 3:,3,0,7\rangle \longrightarrow \langle 1:,3,0,9\rangle \longrightarrow \langle 6:,3,0,9\rangle$$

Maximal execution trace

```
#include <stdio.h>
int main() {
          int x,y;
          printf("Enter an integer: ");
          scanf("%d",&x); y = x;
/* 1: */  while (x != 0) {
             printf("x = %d, y = %d\n",x,y);
/* 2: */     x = x - 1;
/* 3: */     y = y + 2;
          }
/* 4: */  printf("x = %d, y = %d\n",x,y); }
```

```
Enter an integer: 3
x = 3, y = 3
x = 2, y = 5
x = 1, y = 7
x = 0, y = 9
```

```
Enter an integer: -1
x = -1, y = -1
x = -2, y = 1
x = -3, y = 3
x = -4, y = 5
…
x = -738245, y = 1476487
```



initial state $\in$ *init* $[\![P]\!]$ — transition $\in$ *trans* $[\![P]\!]$

$$\langle 1:,3,3,3\rangle \longrightarrow \langle 2:,3,3,3\rangle \longrightarrow \langle 3:,3,2,3\rangle \longrightarrow \langle 1:,3,2,5\rangle \longrightarrow \langle 2:,3,2,5\rangle \longrightarrow \langle 3:,3,1,5\rangle \longrightarrow \langle 1:,3,1,7\rangle \longrightarrow \langle 2:,3,1,7\rangle \longrightarrow \langle 3:,3,0,7\rangle \longrightarrow \langle 1:,3,0,9\rangle \longrightarrow \langle 6:,3,0,9\rangle$$

Maximal trace semantics

- The trace semantics of a program is the set of all possible maximal finite or infinite execution traces for that program

- The trace semantics of a programing language maps programs to their trace semantics

Inductive definition

- Partial traces:
  - A trace with one initial state is a partial trace
  - A partial trace extended by a transition is a partial trace
- Maximal traces:
  - Finite traces with no extension by a transition
  - Infinite traces which prefixes are all partial traces

Fixpoint partial trace semantics

- initial states of program P: $init\,[\![\mathrm{P}]\!]$
- transitions of programs P: $trans\,[\![\mathrm{P}]\!]$
- $\mathrm{F}^{\mathrm{t}}[\![\mathrm{P}]\!]X = \{\ \mathrm{s} \mid \mathrm{s} \in init\,[\![\mathrm{P}]\!]\ \} \cup \{\ \sigma \mathrm{ss}' \mid \sigma \mathrm{s} \in X \wedge \mathrm{ss}' \in trans\,[\![\mathrm{P}]\!]\ \}$
- $\mathrm{S}^{\mathrm{t}}[\![\mathrm{P}]\!] = \mathsf{lfp}^{\subseteq}\ \mathrm{F}^{\mathrm{t}}[\![\mathrm{P}]\!]$

Invariance abstraction

- Collect at each control point the possible values of variables when execution reaches that control point
- $\alpha(X)\mathrm{c} = \{\mathrm{m} \mid \exists \sigma, \sigma'.\ \sigma\langle \mathrm{c}, \mathrm{m}\rangle\sigma' \in X\}$
- Invariance semantics: $\mathrm{S}^{\mathrm{i}}[\![\mathrm{P}]\!] = \alpha(\mathrm{S}^{\mathrm{t}}[\![\mathrm{P}]\!])$

Invariance abstraction

- Collect at each control point the possible values of variables when execution reaches that control point
- $\mathrm{S}^{\mathrm{i}}[\![\mathrm{P}]\!] = \alpha(\mathrm{S}^{\mathrm{t}}[\![\mathrm{P}]\!])\mathrm{c} = \{\mathrm{m} \mid \exists \sigma,\sigma'.\ \sigma\langle \mathrm{c},\mathrm{m}\rangle\sigma' \in \mathrm{S}^{\mathrm{t}}[\![\mathrm{P}]\!]\}$

$\{\langle x_0, x, \mathrm{y}\rangle \mid \mathrm{y} = 3x_0 - 2x\}$ ← /* 1: */

$\{\langle x_0, x, \mathrm{y}\rangle \mid \mathrm{y} = 3x_0 - 2x\}$ ← /* 2: */

$\{\langle x_0, x, \mathrm{y}\rangle \mid \mathrm{y} = 3x_0 - 2x - 2\}$ ← /* 3: */

$\{\langle x_0, x, \mathrm{y}\rangle \mid \mathrm{y} = 3x_0 \wedge x = 0\}$ ← /* 4: */

```
#include <stdio.h>
int main() {
        int x,y;
        printf("Enter an integer: ");
        scanf("%d",&x); y = x;
/* 1: */ while (x != 0) {
           printf("x = %d, y = %d\n",x,y);
/* 2: */    x = x - 1;
/* 3: */    y = y + 2;
        }
/* 4: */ printf("x = %d, y = %d\n",x,y); }
```

Calculations design of the verification conditions

- $\alpha(\mathrm{F}^{\mathrm{t}}[\![\mathrm{P}]\!]X)$
  $= \boldsymbol{\lambda}\mathrm{c}.\{\mathrm{m} \mid \exists\sigma,\sigma'.\ \sigma\langle\mathrm{c},\mathrm{m}\rangle\sigma' \in X\}$
  $= \ldots$
  $= \mathrm{F}^{\mathrm{i}}[\![\mathrm{P}]\!](\alpha(X))$

  where $\mathrm{F}^{\mathrm{i}}[\![\mathrm{P}]\!]$ are the Turing/Floyd/Naur/Hoare verification conditions

- It follows that $\mathrm{S}^{\mathrm{i}}[\![\mathrm{P}]\!] = \mathrm{lfp}^{\dot{\subseteq}}\ \mathrm{F}^{\mathrm{i}}[\![\mathrm{P}]\!]$

- The proof method is then by fixpoint induction (Tarski 1955)

Application to the semantics of programming languages

General idea

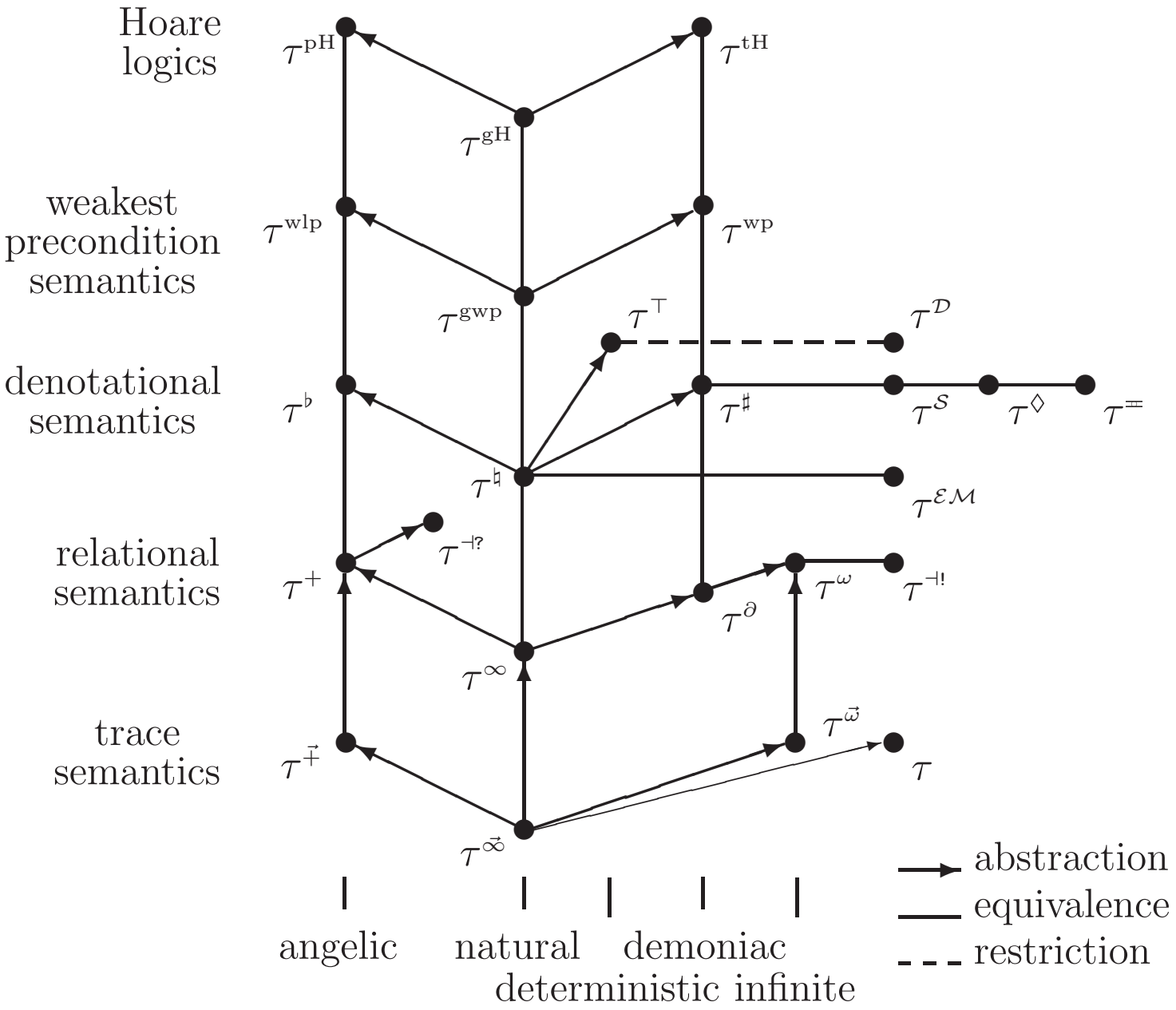
- All known semantics are abstractions of a most precise semantics

Abstraction to denotational semantics

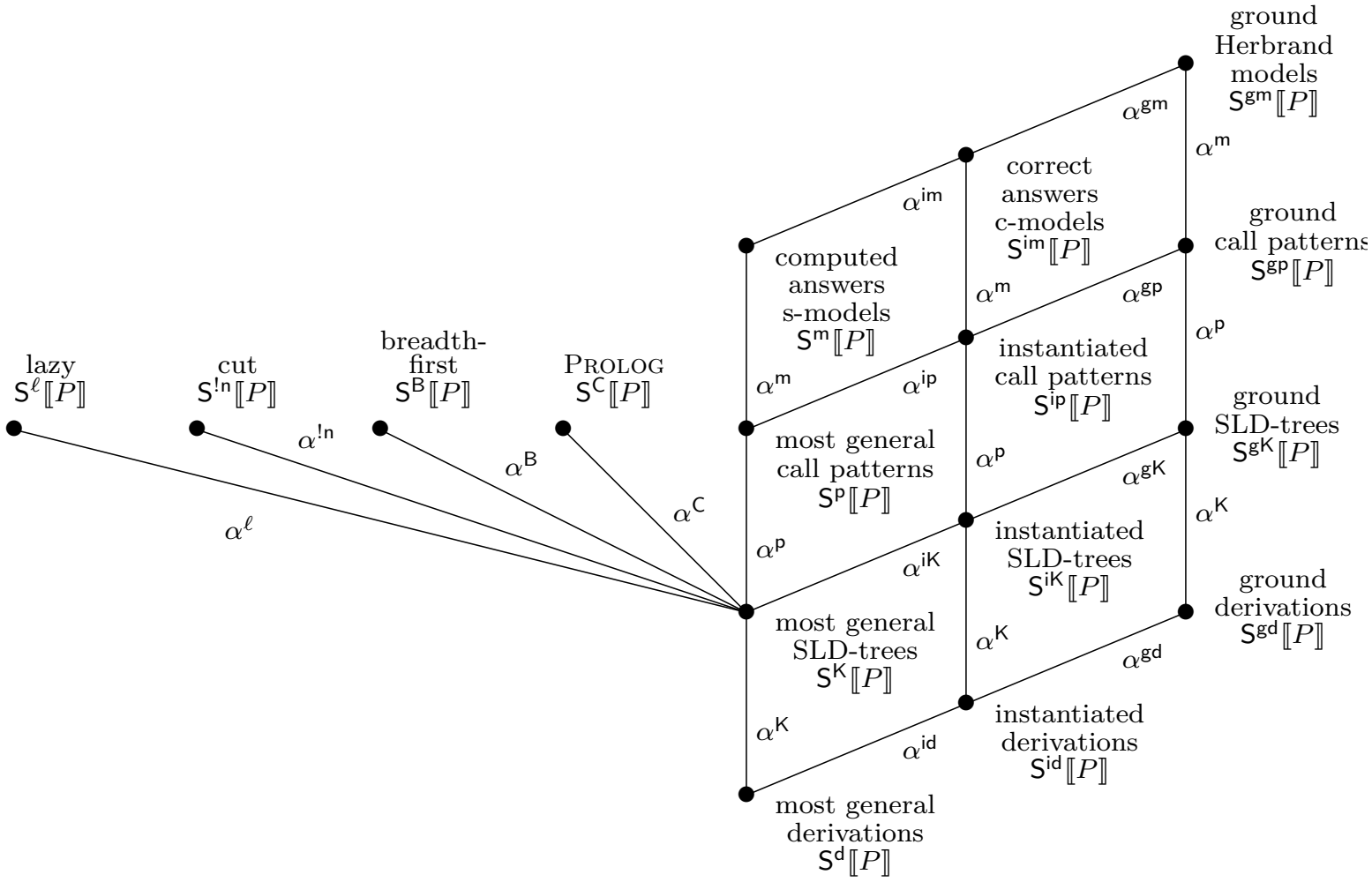
- The maximal trace semantics $\mathrm{S}^{\mathrm{m}}[\![\mathrm{P}]\!]$ (maximal finite and infinite execution traces
- Denotational semantics abstraction:
  - $\mathrm{S}^{\mathrm{d}}[\![\mathrm{P}]\!] = \alpha(\mathrm{S}^{\mathrm{m}}[\![\mathrm{P}]\!])$
  - $\alpha(X) = \boldsymbol{\lambda}\mathrm{s}.\{\mathrm{s}' \mid \exists \sigma.\ \mathrm{s}\sigma\mathrm{s}' \in X\} \cup \{\bot \mid \exists \sigma.\ \mathrm{s}\sigma\ldots \in X\}$

  i.e. a map of initial states to the set of final states plus $\bot$ in case of non-termination

Hierarchy of abstractions



idem for Prolog



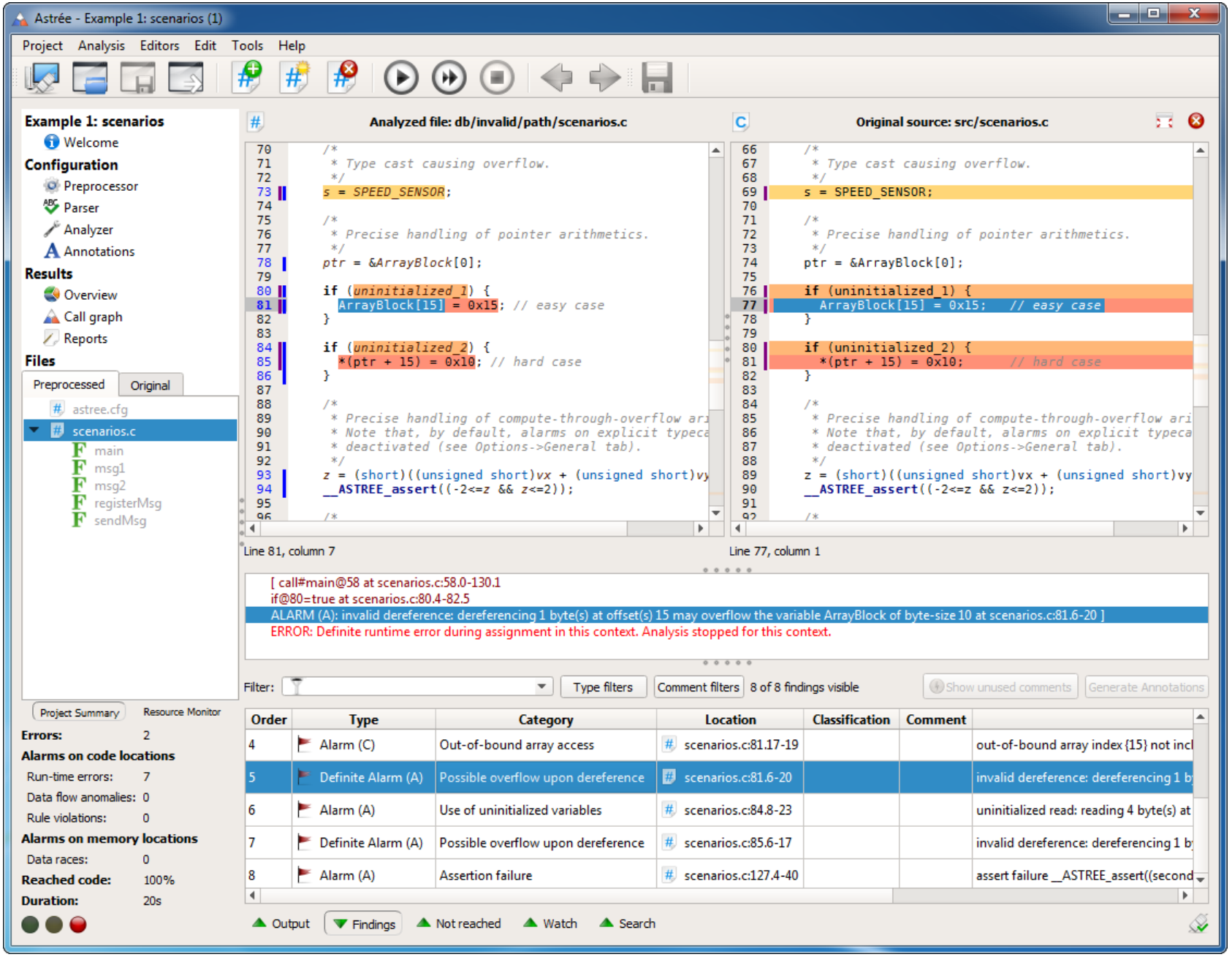
- all semantics are abstractions of $\mathsf{S}^{d}[\![P]\!]$

Conclusion

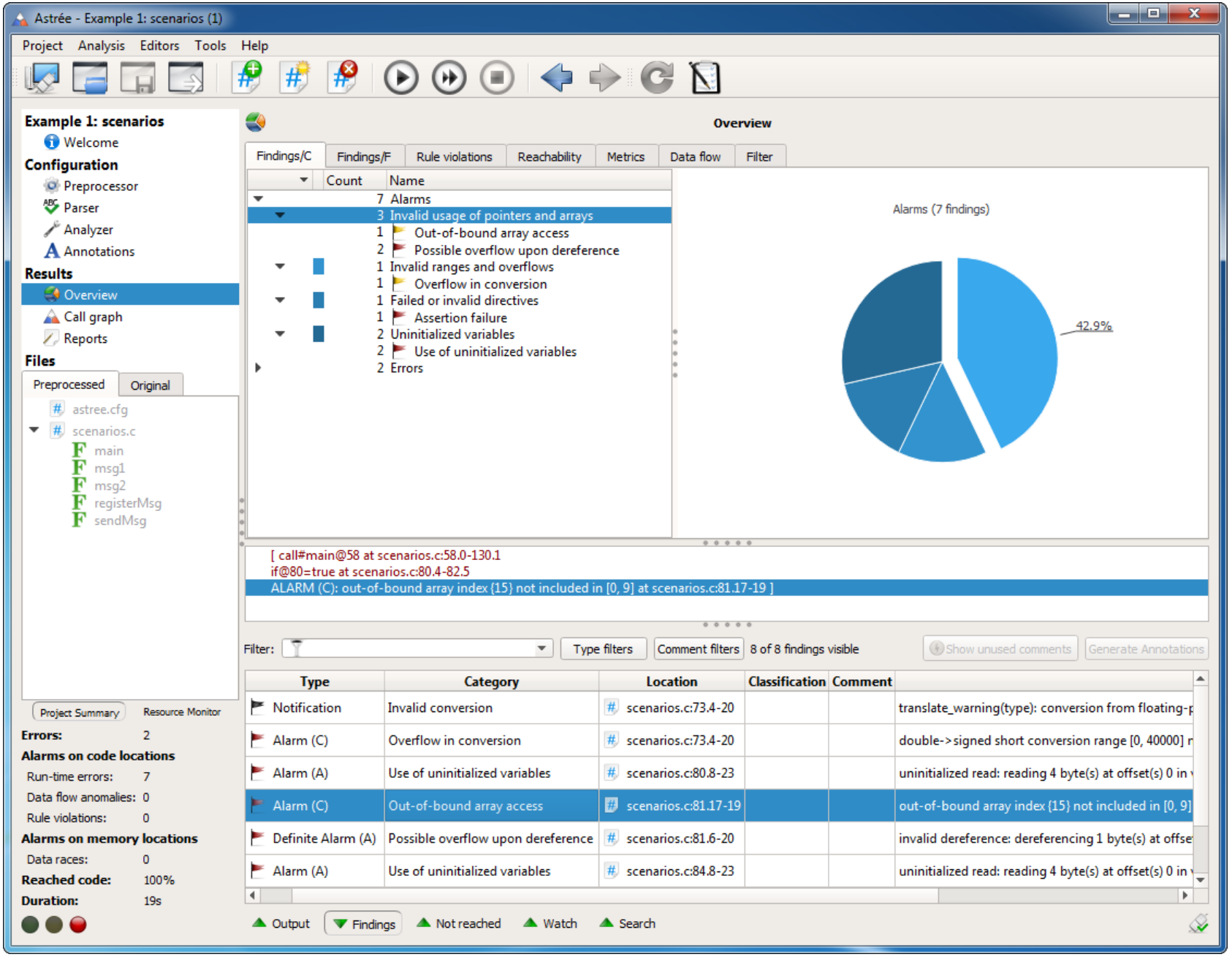
Abstract interpretation

- A well-developed theory, still in progress
- Active research e.g.
  - abstract domains to handle e.g. complex data structures
  - abstraction of parallelism with weak memory models
  - applications to biology, …
- Industrial-quality static analyzers

Industrialisation: Astrée



Industrialisation: Astrée



Many other static analyzers

- Julia (Java) http://www.juliasoft.com
- Ikos, NASA https://ti.arc.nasa.gov/opensource/ikos/
- Clousot for code contract, Microsoft, https://github.com/Microsoft/CodeContracts
- Infer (Facebook) http://fbinfer.com
- Zoncolan (Facebook)
- Google
- …

Static analysis for software development

- Users of Astrée:

- Why not all software developers use static analysis tools?

Irresponsibility

- Computer engineering is the only technology where developers are not responsible for their errors, even the trivial ones:

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The future

- Safety and security does matter to the general public
- Computer scientists will ultimately be held responsible for there errors
- At least the automatically discoverable ones
- Since this is now part of the state of the art
- Automatic static analysis, verification, etc has a brilliant future.

Francesco Logozzo, designer of the Zoncolan static analyzer at Facebook wrote me on 09/12/2016:

``Finding people who really know static analysis is very hard, you should tell your students that if they want a great job in a Silicon Valley company they should study abstract interpretation not JavaScript. Feel free to quote me on that ;-)”

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The End, Thank You