On Verifying Causal Consistency

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Strong (sequential) consistency



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 Strong (sequential) consistency is impossible while being available and tolerating network partitions: the CAP theorem ¹



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• Tolerating **faults** while preserving **availability** leads to **anomalies** w.r.t. strong (sequential) consistency



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Updates are seen in different orders

Goal: Verifying Causal Consistency

The set of allowed anomalies are defined by **weak consistency** criteria, e.g., eventual consistency, causal consistency.

Algorithmic methods for checking causal consistency.

Single-Trace Verification: Check if one trace is causally consistent
Application to testing, monitoring (by enumerating traces)

All-Traces Verification: Check if all traces are causally consistent

Static verification

Single-Trace Verification:

• NP-complete for most consistency criteria²

 ³Memory Model-aware Testing. Furbach et al. 2014.
 ⁴Model-Checking of Correctness Conditions. Alur et al. 1996.
 ⁵On the complexity of linearizability. H. 2015.
 ⁶Verifying Eventual Consistency of ORS. Bouajjani et al. 2014.

Single-Trace Verification:

- NP-complete for most consistency criteria²
- NP-complete for causal consistency as well

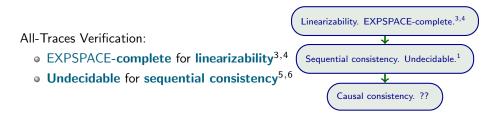
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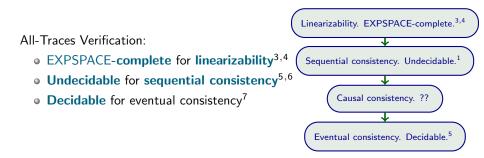
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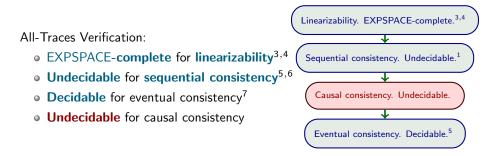
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- For key-value stores
- For a bounded number of sites
- For finite-state implementations
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(Input: finite-state automaton representing all traces)

Key Observation: Implementations Are Data Independent

Key-value store implementations are data independent

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X contains **4-6** bad patterns

Results: Complexity/Decidability and Reduction to Reachability

Bad patterns implications for data-independent implementations:

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Checking whether all traces of a data-independent finite-state implementation are causally consistent is decidable.

• Definition(s) of causal consistency

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- Characterize all causal consistency violations using bad patterns

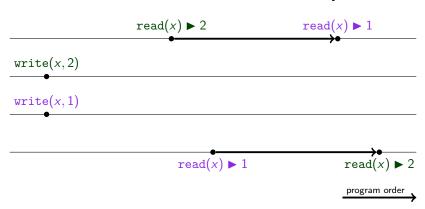
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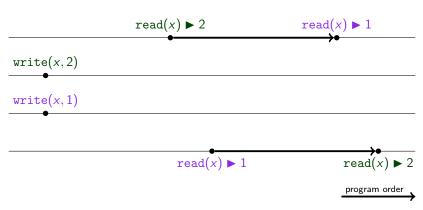
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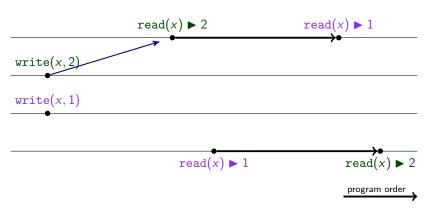


There exists a **causality order** CO such that

the causal past of every read can explain its value

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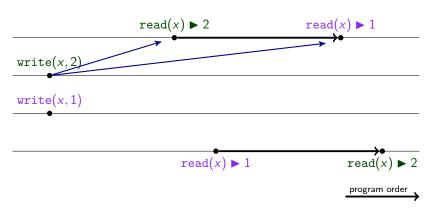


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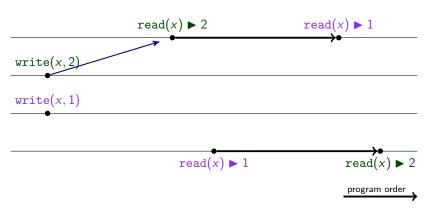


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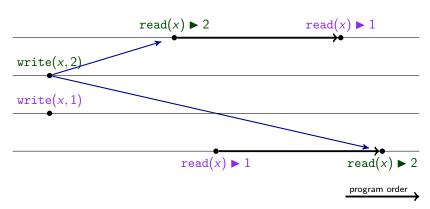


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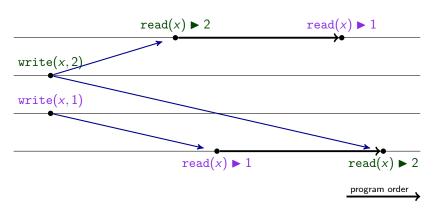


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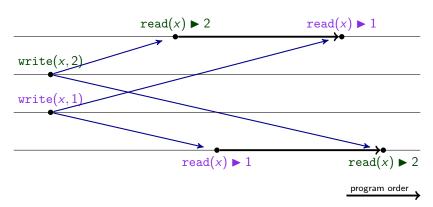


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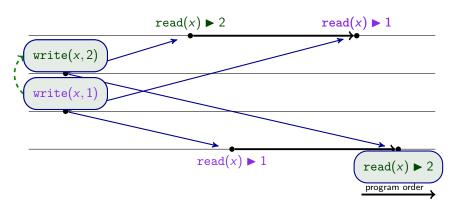


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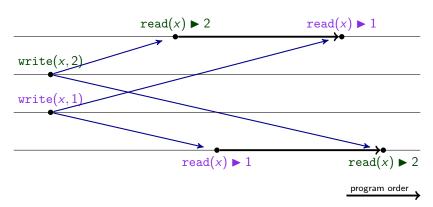


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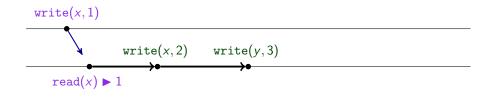
Definition of Causal Consistency $read(x) \ge 1$ $read(x) \ge 2$ write(x, 2)write(x, 1) $read(x) \ge 1$ $read(x) \ge 2$ program order

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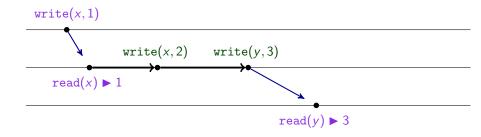
Causal Consistency Violations

Causally related writes must be seen by all sites in the same order.



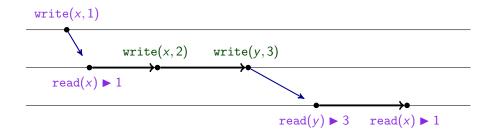
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AXCAUSALVALUE : $\forall o \in O$. CausalPast(CO, o) $\sqsubseteq S$

Formalizing Causal Consistency

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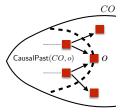
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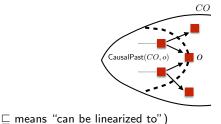
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Causal Convergence⁸

- Conflicts are resolved using a global arbitration order
- Strong eventual consistency:

If two sites see the same writes, they are in the same state⁷

⁷A comprehensive study of CRDTs. 2011. Shapiro et al.

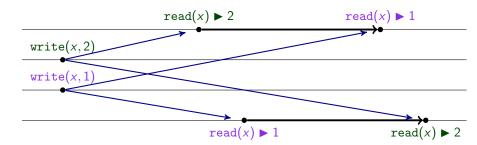
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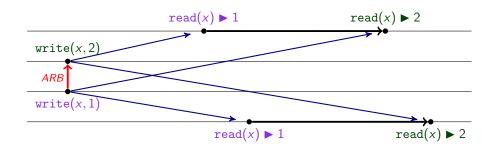
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"
 ARB" means adding the constraints in ARB)

Satisfying Causal Convergence

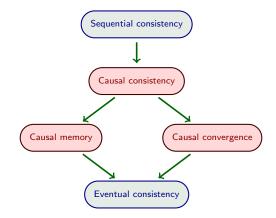


Causal Consistency

Satisfying Causal Convergence but not Sequential Consistency



Different Notions of Causal Consistency



Causal memory = Causal consistency + local arbitration

Outline

• Definition(s) of causal consistency

• Characterize all causal consistency violations using bad patterns

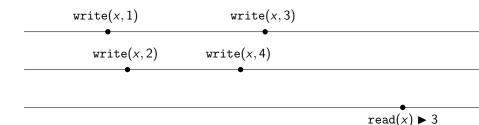
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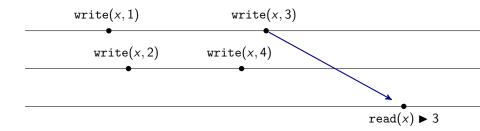
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 \Rightarrow We can assume written values are unique.

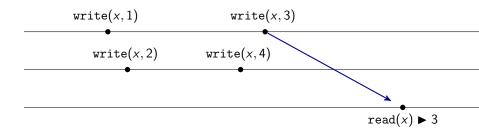
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Unicity of writes implies a canonical **causality** relation (included in every other causality relation).

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- PO (program order): connects operations from the same site
- RF (reads-from relation): connects write to read
- CO (causal order): defined as $(PO \cup RF)^+$

Bad Pattern for Causal Consistency: WriteCORead

- Two writes w_1 and w_2 , and one read r_1 on the same variable:
 - r₁ reads-from w₁
 - w₁ <_co w₂ <_co r₁

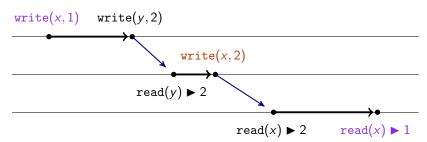
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Example:



Data Independent Implementations Bad Patterns

WriteCORead: Litmus tests

```
W1 <PO W2 <PO T1:
Write(x, 1)
Write(x, 2)
read(x) ▶ 1</pre>
```

WriteCORead: Litmus tests

$w_1 <_{PO} w_2 <_{PO} r_1$:	$w_1 <_{PO} w_2 <_{CO} r_1$:	
write(x, 1) write(x, 2) read(x) \triangleright 1	<pre>write(x, 1) write(x, 2) write(y, 3)</pre>	$read(y) \triangleright 3$ $read(x) \triangleright 1$

write(y, 3) ||

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w 1 < co	$w_2 <_{PO} r_1$:	$w_1 <_{CO} w_2 <_{CO}$	o r ₁ :	
write($(x,1)$ read $(y) \triangleright 3$	write(x,1)	$read(y) \triangleright 3$	read(z

$read(y) \triangleright 3$	write(x,1)	$read(y) \triangleright 3$	$read(z) \triangleright 4$
write(x, 2)	write(y, 3)	write $(x, 2)$	$\texttt{read}(x) \blacktriangleright 1$
$\texttt{read}(x) \blacktriangleright 1$		write(z, 4)	

Bad Patterns for Causal Consistency

- WriteCORead: two writes w_1 and w_2 , and one read r_1 on some x s.t.
 - r1 reads-from w1
 - w₁ <_{CO} w₂ <_{CO} r₁
- CyclicCO: $CO = (PO \cup RF)^+$ is cyclic
- ThinAir: a read operation r = read(x) ► v with v ≠ 0 s.t.
 w ≮_{RF} r for every write w
- WriteCOInit: a read operation r = read(x) ► 0 s.t.
 w <_{CO} r for some write w on x

Bad Patterns for Causal Consistency Variants

Causal Consistency	Causal Memory	Causal Convergence
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Theorem (Bad Patterns)

A trace doesn't satisfy the criterion X iff it contains a bad pattern for X.

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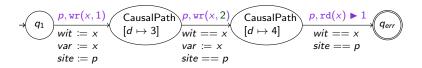
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Singe-Trace Verification of causal consistency is **polynomial** when **writes are unique**.

(By checking the absence of bad patterns.)

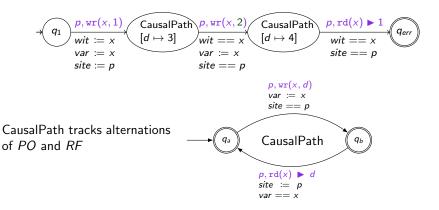
Recognizing Bad Patterns with Register Automata

- By data independence, we can use a bounded number of values
- Registers are needed to store variable names while tracking causality paths
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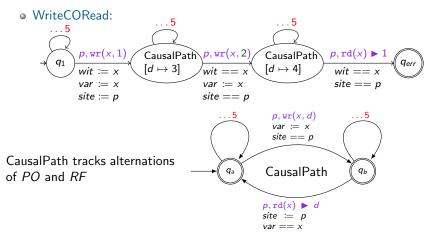
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Machine M tracking all bad patterns.

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- Manual or semi-automated proofs

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Future work:

- Bad patterns for other criteria (FIFO consistency, ...)
- for other specifications (Multi-Value Register, CRDTs, ...)

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- Bad patterns for other criteria (FIFO consistency, ...)
- for **other specifications** (Multi-Value Register, CRDTs, ...)
- Application to existing causally consistent systems to prove their correctness (or find bugs)

Summary:

- Difficult to verify causal consistency in general (Single-Trace: NP-complete, All-Traces: Undecidable)
- Bad patterns for data-independent implementations
 - Single-Trace: PTime, All-Traces: Decidable
 - Polynomial-time reduction to reachability: approach for verifying causal consistency

Future work:

- Bad patterns for other criteria (FIFO consistency, ...)
- for **other specifications** (Multi-Value Register, CRDTs, ...)
- Application to existing causally consistent systems to prove their correctness (or find bugs)

Thank you