Progressive Wasserstein Barycenters of Persistence Diagrams
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Additional Materials

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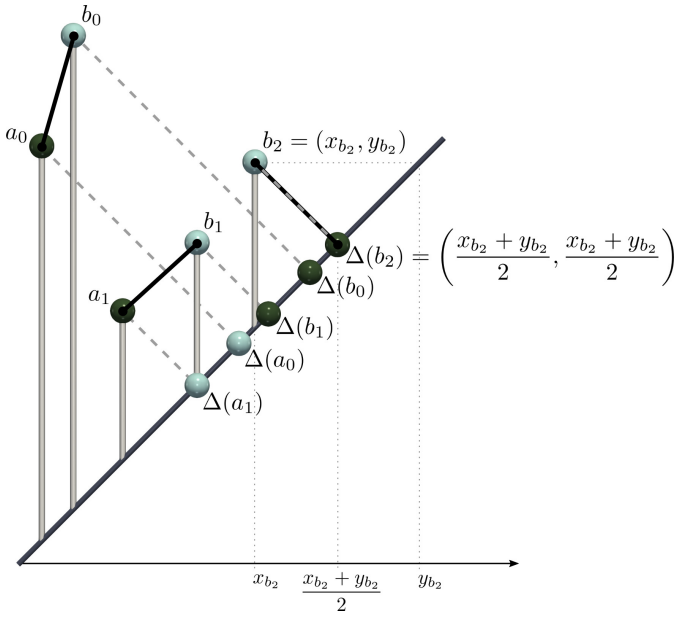

Fig. 1. Illustration for section 2 : Example of a diagonal matching in the assignment between two diagrams (green and cyan spheres) of different cardinality. The *augmented* diagrams are constructed by including the orthogonal projections $\Delta(a)$ (grey dashed lines) of each point a onto the diagonal of the other diagram. The assignment problem thus becomes balanced (matchings are shown with black lines). This allows a persistence pair to be matched to a diagonal point (b_2 and $\Delta(b_2)$). Diagonal points of different diagrams can be matched together with zero cost.

Table 1. Wasserstein distances (column W_2) between the barycenters computed with our approach and the *Auction* approach [2]+[1], for all ensembles. For comparison, the maximal and minimal distances observed between diagrams for each ensemble are also reported. Note that the two algorithms (our progressive method and the *Auction* approach) may converge to different local minimas of the Fréchet energy, which can be arbitrarily distant from each other while being of similar quality (visually and quantitatively in terms of Fréchet energy). Overall, the distances between the two outputs are comparable to the minimum distances observed within each ensemble. This indicates that the barycenters resulting from the two approaches are rather close, in particular relatively to the maximum observed distances.

Data set	N	$\#_{\mathscr{D}(f_i)}$	Maximal distance	Minimal distance	W_2
Gaussians	100	2,078	5.30	1.52	0.75
Vortex Street	12	36	25.07	0.02	1.38
Starting Vortex	45	14	471.52	9.27	25.29
Isabel (3D)	12	1,337	72.96	19.84	15.79
Sea Surface Height	48	1,379	2.18	0.76	0.69

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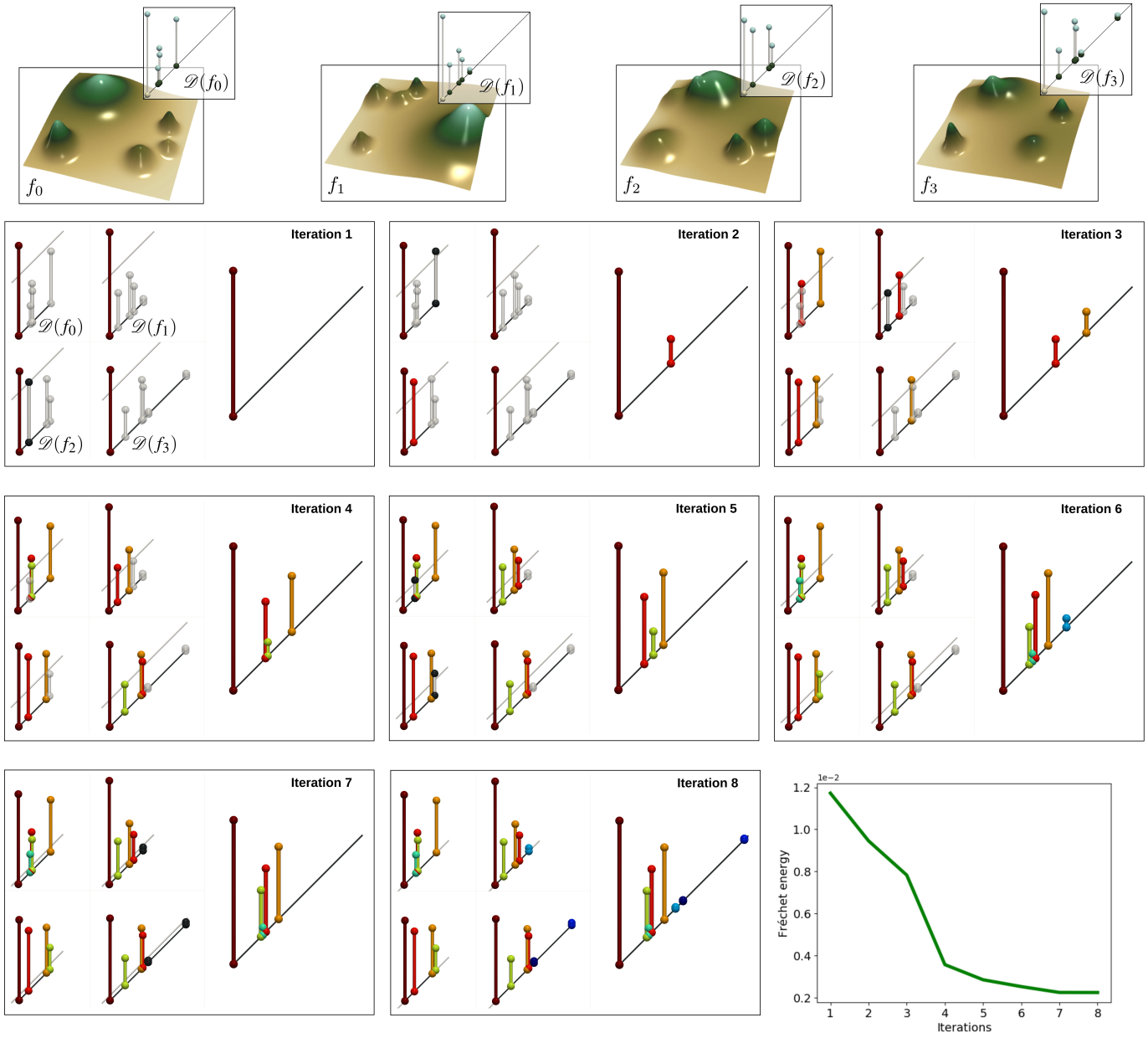

Fig. 2. Illustration for the section 3 : Iterations of our progressive algorithm on a randomly generated toy example. The top row presents the input toy ensemble and the corresponding persistence diagrams. Then, for each iteration, the four input diagrams are showed on the left and the corresponding barycenter on the right. In the input diagrams, under the persistence threshold (white line), the persistence pairs in transparent white have not been added yet to the assignment problem. The colors of the remaining pairs indicate their matched pairs in the barycenter. Initially, only the largest pair is present in each diagram, and the barycenter is identical to the first thresholded input diagram. The persistence threshold is progressively decreased along the relaxation process, allowing more persistence pairs to be added in the diagrams. At each iteration, one *Update* step occurs, followed by a decreasing of the persistence threshold, and one *Assignment* step, whose result is displayed. As such, persistence pairs in the input diagrams can be matched to the diagonal, and are in this case represented with a white bar with black spheres. This event takes place at iterations 1, 2, 3, 5, and 7, and always precedes the addition of a pair in the barycenter. On the contrary, pairs can be deleted from the barycenter if they are not matched to any pair anymore in the input diagrams, such as the blue pair at iteration 6. The evolution of the Fréchet energy (bottom right) illustrates the fast convergence of the barycenter towards a local minimizer.

REFERENCES

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[2] K. Turner, Y. Mileyko, S. Mukherjee, and J. Harer. Fréchet Means for Distributions of Persistence Diagrams. *Disc. Compu. Geom.*, 2014.